ST.ANNE'S COLLEGE OF ENGINEERING AND TECHNOLOGY

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(An ISO 9001: 2015 Certified Institution)

ANGUCHETTYPALAYAM, PANRUTI – 607 106.

DEPARTMENT OF MECHANICAL ENGINEERING

ME 86594- DYNAMICS OF MACHINERY

THIRD YEAR - FIFTH SEMESTER

PREPARED BY

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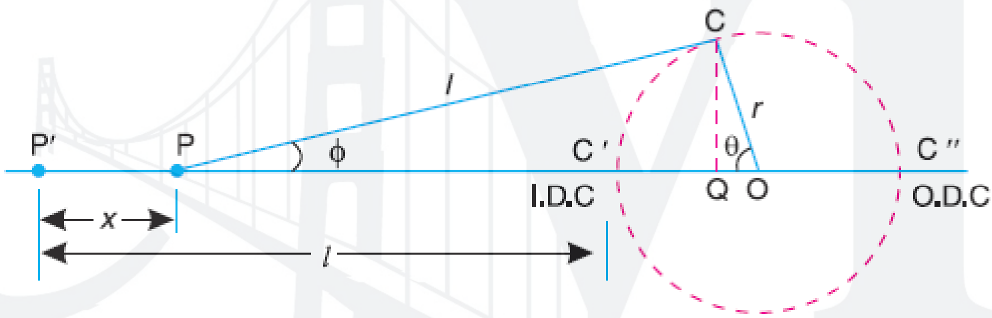
UNIT I FORCE ANALYSIS

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle –Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Camfollower mechanism.

Analytical Method for Velocity and Acceleration of the Piston

Consider the motion of a crank and connecting rod of a reciprocating steam engine as shownin Fig. 15.7. Let *OC* be the crank and *PC* the connecting rod. Let the crank rotates with angularvelocity of ω rad/s and the crank turns through an angle θ from the inner dead centre (brieflywrittenas I.D.C). Let *x* be the displacement of a reciprocating body *P* from I.D.C. after time *t*seconds, duringwhich the crank has turned through an angle θ.



Let l = Length of connecting rod between the centres,

r = Radius of crank or crank pin circle,

θ = Angle of crank from Crank

φ = Inclination of connecting rod to the line of stroke *PO*, and

n = Ratio of length of connecting rod to the radius of crank = l/r.

Velocity of the piston

$x = P'P = OP' - OP = (P'C' + C'O) - (PQ + QO)$

$= (l + r) - (l \cos \varphi + r \cos \theta)$ $(PQ = l\cos\varphi)$.

$(QO = r\cos\theta)$

$= l + r - l\cos\varphi - r\cos\theta = r(1 - \cos\theta) + l(1 - \cos\varphi)$

$$=r\,[(1-\cos\theta) + \frac{l}{r}(1-\cos\varphi)]$$

From triangles *CPQ* and *CQO*

$CQ = l \sin\varphi = r \sin\theta$ or $l/r = \sin\theta/\sin\varphi$

$n = \sin\theta/\sin\varphi$ or $\sin\varphi = \sin\theta/n$

We know that, $\cos\varphi = (1-\sin^2\varphi)^{\frac{1}{2}} = \left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}$

Expanding the above expression by binomial theorem, we get

$$\text{Cos}\varphi = 1 - \frac{1}{2}\frac{\sin^2\theta}{n^2} + \cdots$$

$$1 - \cos\varphi = \frac{\sin^2\theta}{2n^2}$$

Substituting the value of $(1 - \cos\varphi)$ in equation **(*i*),** we have

$$x = r\,[(1 - \cos\theta) + n \times \frac{\sin^2\theta}{2n^2}] = r[(1 - \cos\theta) + \frac{\sin^2\theta}{2n}]$$

Differentiating the above equation with respect to θ,

$$\frac{dx}{dt} = r\,[\sin\theta + \frac{\sin 2\theta}{2n}]$$

$$v_{PO} = v_P = \frac{dx}{dt} = \frac{dx}{d\theta}\,\frac{d\theta}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

$$v_{PO} = v_P = r\omega\,[\sin\theta + \frac{\sin 2\theta}{2n}]$$

Acceleration of the Piston

$$a_p = \frac{dv_p}{dt} = \frac{dv_p}{d\theta}\,\frac{d\theta}{dt}$$

$$a_p = r\omega^2\,[\cos\theta + \frac{\cos 2\theta}{n}]$$

Angular Velocity of the Connecting Rod

$$CQ = l\,sin\phi = r sin\theta$$

$$sin\phi = \frac{r}{l} sin\theta = \frac{sin\theta}{n}$$

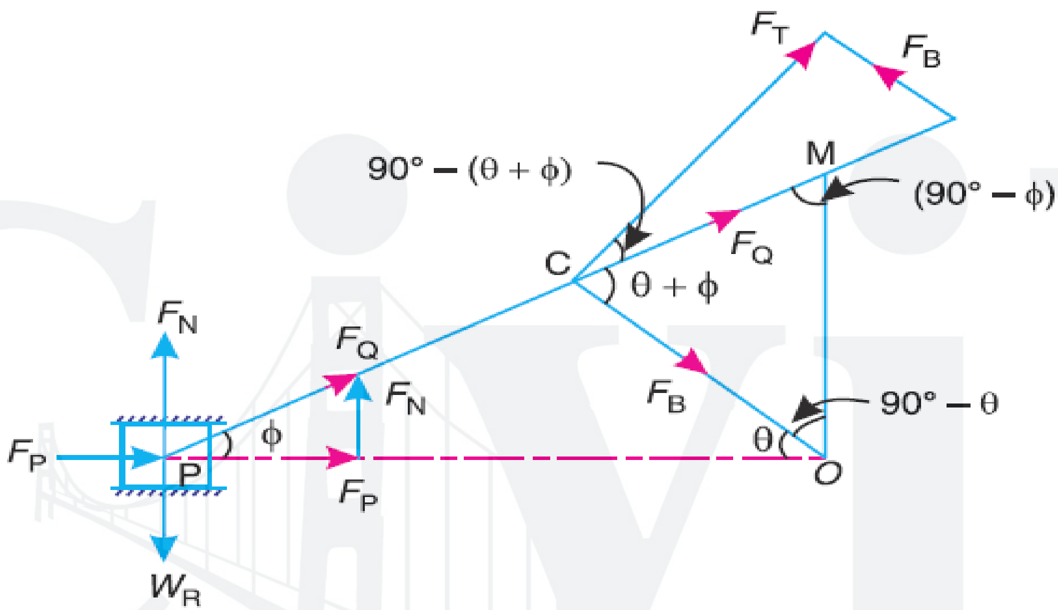
$$\cos\phi\,\frac{d\phi}{dt} = \frac{cos\theta}{n}\,\frac{d\theta}{dt} = \frac{cos\theta}{n}\,\omega$$

$$\omega_{PC} = \frac{\omega cos\theta}{(n^2 - \sin^2\theta)^{\frac{1}{2}}}$$

Angular Aceeleration of the connecting Rod

$$\alpha_{PC} = \frac{-\omega^2 si(n^2 - 1)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}$$

FORCES ON THE RECIPROCATING PARTS OF AN ENGINE



Piston Effort

F_p = Net load on the piston ± Inertia force

For Horizontal

$F_p = F_L \pm F_I$ (Negelecting frictional resistance)

$F_p = F_L \pm F_I - R_F$ (Considering frictional resistance)

For Vertical

$F_p = F_L \pm F_I \mp W_R$ (Negelecting frictional resistance)

$F_p = F_L \pm F_I - R_F$ (Considering frictional resistance)

Load on the piston

$F_L = p\frac{\pi}{4}d^2$ For Single acting

$F_L = p_1A_1 - p_2A_2$ For Double acting

Force acting along the connecting rod

$$F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

Thrust on the slides of Cylinder walls

$$F_N = F_P tan\varphi$$

Crank pin effort

$$F_T = \frac{F_P}{cos\varphi}\sin(\theta + \varphi)$$

Thrust on the bearing

$$F_B = \frac{F_P}{cos\varphi}\cos(\theta + \varphi)$$

Crank effort

$$T = F_T \, x r$$

$$T = F_P \left[sin\theta + \frac{cos\theta sin\theta}{\sqrt{n^2 - \sin^2\theta}}\right] r$$

$$T = F_P \left[sin\theta + \frac{sin2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] r$$

1. **If the crank and the connecting rod are 300 mm and 1 m long respectively and the crank rotates at a constant speed of 200 r.p.m., determine: 1. The crank angle at which the maximum velocity occurs, and 2. Maximum velocity of the piston.**

Solution. Given : r = 300 mm = 0.3 m ; l = 1 m ; N = 200 r.p.m. or

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 200}{60} = 20.94 rad/s$$

To Find.

1. The crank angle at which the maximum velocity occurs(ɵ) and

2. Maximum velocity of the piston.

1. ***Crank angle at which the maximum velocity occurs***

Let θ = Crank angle from the inner dead centre at which the maximum velocity occurs.

We know that ratio of length of connecting rod to crank radius,

$n = l/r = 1/0.3 = 3.33$

and velocity of the piston,

$$v_{PO} = v_P = r\omega\,[\sin\theta + \frac{\sin 2\theta}{2n}]$$

For Maximum Velocity of the piston

$$\frac{dv_p}{d\theta} = 0$$

$$r\omega\,[\cos\theta + \frac{\cos 2\theta}{n}] = 0$$

$$ncos\theta + 2cos^2\theta - 1 = 0$$

$$3.33cos\theta + 2cos^2\theta - 1 = 0$$

$$cos\theta = 0.26$$

$$\theta = 75°$$

2. Maximum velocity of the piston

Substituting the value of θ = 75° in equation (i), maximum velocity of the piston,

$$v_{Pmax} = r\omega\,[\sin\theta + \frac{\sin 2\theta}{2n}]$$

$$v_{Pmax} = r\omega\,[\sin 75 + \frac{\sin 150}{2(3.33)}]$$

$$v_{Pmax} = 6.54 m/s$$

2. The Lengths of crank and connecting rod of a horizontal engine are 200mm and 1m respectively. The crank is rotating at 400rpm. When the crank has turned through 30° from the inner dead centre, the difference of pressure between cover and piston rod is 0.4N/mm^2. If the mass of the reciprocating parts is 100kg and cylinder bore is 0.4m,then calculate: (i) inertia force, (ii) force on piston, (iii) piston effort, (iv) thrust on the sides of the cylinder walls, (v) thrust in the connecting rod, and (vi) crank effort.

Given

r = 200mm, l = 1m, N = 400rpm, $\theta = 30°$, p1-p2 = 0.4N/mm^2 =0.4x10^6 N/m^2,

m_R = 100kg, D = 0.4m

Solution

n = l/r = 1/0.2 = 5

$$\omega = \frac{2\pi N}{60} = \frac{2(400)}{60} = 41.89\, rad/s$$

(i) Inertia force

$$F_I = m_R \; r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$F_I = 100.(0.2)(41.89)^2 \left[\cos 30 + \frac{\cos 60}{5}\right]$$

$$F_I = 33.903KN$$

(ii) Force on piston

$$F_L = p_1A_1 - p_2A_2 = (p_1 - p_2)$$

$$F_L = 0.4 \; x \; 10^6 \frac{\pi}{4}(0.4)^2 = 50.265KN$$

(iii) Piston Effort

$$F_p = F_L - F_I = 16.36KN$$

(iv) thrust on the sides of the cylinder walls

$$sin\phi = \frac{r}{l} sin\theta = \frac{sin\theta}{n} = \frac{sin30}{5} = 0.1$$

$$\varphi = 5.74°$$

$$F_N = F_P tan\varphi = 16.36 \tan(5.74) = 1.644KN$$

(v) Thrust in the connecting rod

$$F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}} = 16.44KN$$

(vi) Crank effort

$$T = F_T \text{ x } r$$

$$F_T = \frac{F_P}{cos\varphi} \sin(\theta + \varphi) = 9.605KN$$

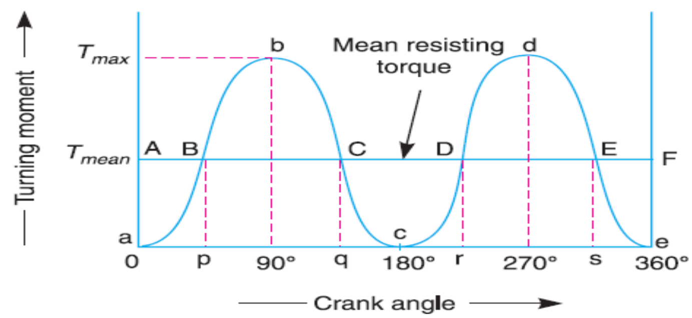
$$T = 9.605 \text{ x } 0.2 = 1921.13KN$$

TURNING MOMENT DIAGRAMS AND FLYWHEELS

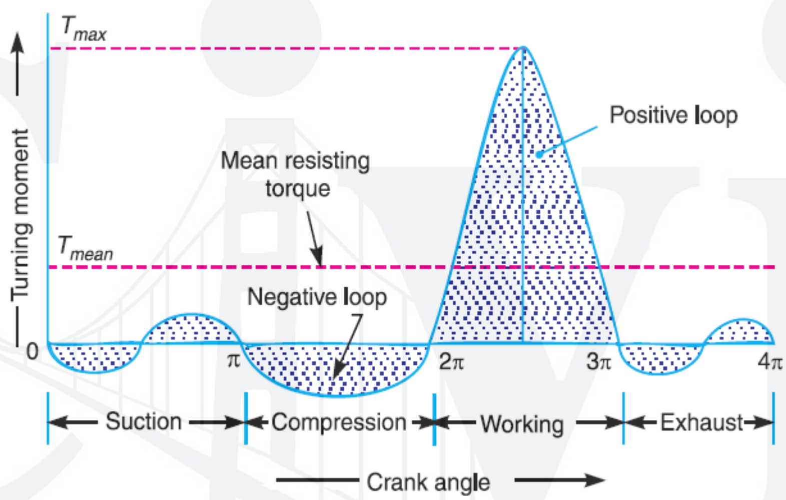
Turning Moment Diagrams

The turning moment diagram is the graphical representation of the turning moment(T) for various positions of the crank(θ).

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.



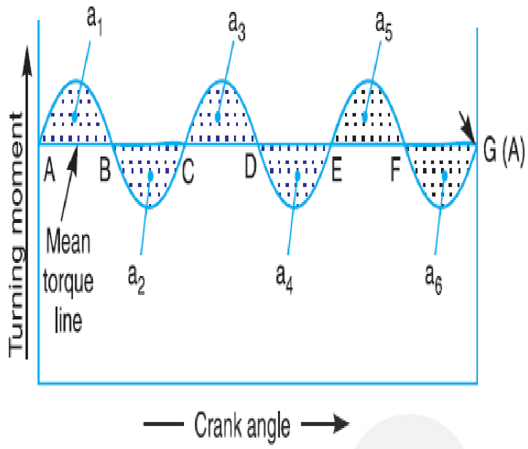
Turning Moment Diagram for a single cylinder Four stroke I C Engine



A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4 πradians).

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig.

Maximum Fluctuation of Energy



ΔE = Maximum Energy – Minimum Energy

Energy Stored in Flywheel

$$\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$$

Coefficient of fluctuation of energy (C_E)

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone per cycle}}$$

Maximum Fluctuation of Speed

ΔS = Maximum Speed – Minimum Speed

$$C_S = \frac{\text{Maximum fluctuation of speed}}{\text{Mean Speed}}$$

$$C_s = \frac{2(N_1 + N_2)}{N_1 + N_2}$$

Coefficient of Steadiness

$$m = \frac{1}{C_s}$$

$$Workdone\ per\ cycle = T_{mean}.\theta = \frac{P\ 60}{n}$$

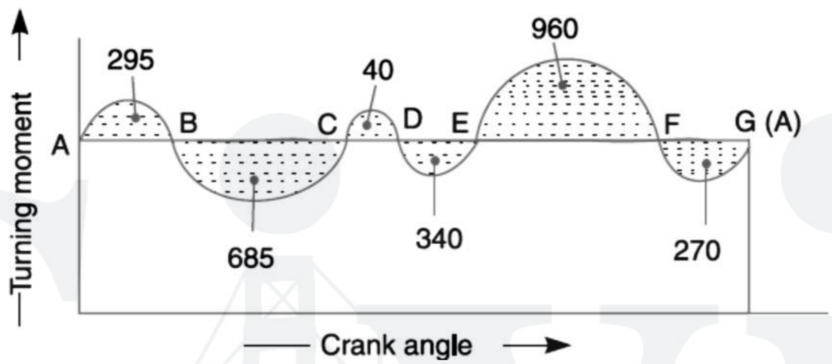
Where,

n = number of working strokes per minute.

$$T_{mean} = \frac{P\ 60}{2\pi N}$$

Crank position	Flywheel energy
A	E
B	$E+a_1$
C	$E+a_1- a_2$
D	$E+a_1- a_2 +a_3$
E	$E+a_1- a_2 +a_3- a_4$
F	$E+a_1- a_2 +a_3- a_4 +a_5$
G	$E+a_1- a_2 +a_3- a_4 +a_5- a_6$

The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 5 N-m ; crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 r.p.m.



Given

1mm = 5 N-m, 1mm = 1º, m = 36kg,, k = 150mm, N = 1800 rpm

$$1mm2 \ = \ 5 \ \times 1 \times \frac{\pi}{180} = \ \ 0.0877$$

Crank position	Flywheel energy	Energy (mm^2)
A	E	E
B	E+295	E+295 (Max.E)
C	E+295-685	E-390
D	E+295-685+40	E-350

$$\omega = \ \frac{2\pi N}{60} = \frac{2(1800)}{60} = 188.52$$

Solution

E	E+295-685+40-340	E-690 (Min.E)
F	E+295-685+40-340+960	E+270
G	E+295-685+40-340+960-270	E

$$\Delta E = Maximum\ energy - Minimum\ energy$$

$$\Delta E = (E + 295) - (E - 690) = 985\ mm^2 = 86\ N - m$$

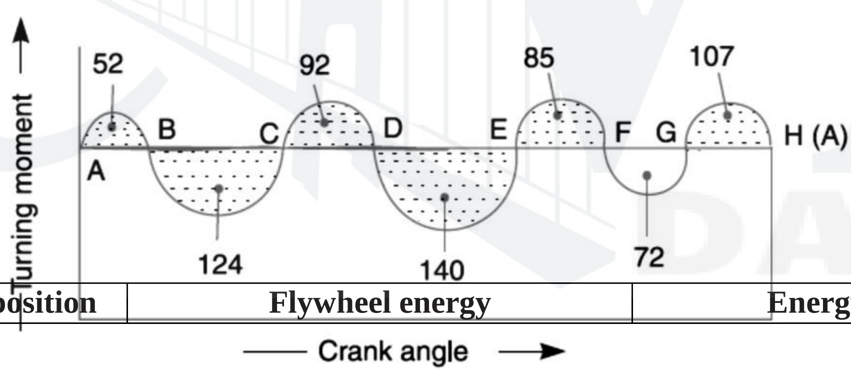
C_s = coefficient of fluctuation of speed

$$\Delta E = mk^2\omega^2 C_s = 36.(0.15)^2(188.52)^2 C_s$$

$$28.787\ C_s = 86$$

$$C_s = 0.003 = 0\ .3\%$$

The turning moment diagram for a multicylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows : + 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm2, when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.



Crank position	Flywheel energy	Energy (mm^2)

Given

1mm = 600 N-m, 1mm = 3º, C_s = ±1.5% = 3% = 0.03, k = 0.5m, N = 600 rpm

$$1mm2 = 600 \times 3 \times \frac{\pi}{180} = 31.4$$

$$\omega = \frac{2\pi N}{60} = \frac{2(1800)}{60} = 62.54$$

A	E	E
B	E+52	E+52 (Max.E)
C	E+52-124	E-72
D	E+52-124+92	E+20
E	E+52-124+92-140	E-120 (Min.E)
F	E+52-124+92-140+85	E-35
G	E+52-124+92-140+85-72	E-107
H	E+52-124+92-140+85-72+107	E

Solution

$$\Delta E = Maximum\ energy - Minimum\ energy$$

$$\Delta E = (E + 52) - (E - 120) = 172\ mm^2 = 5404\ N - m$$

C_s = coefficient of fluctuation of speed

$$\Delta E = mk^2\omega^2 C_s = m.(0.5)^2(62.54)^2 0.03$$

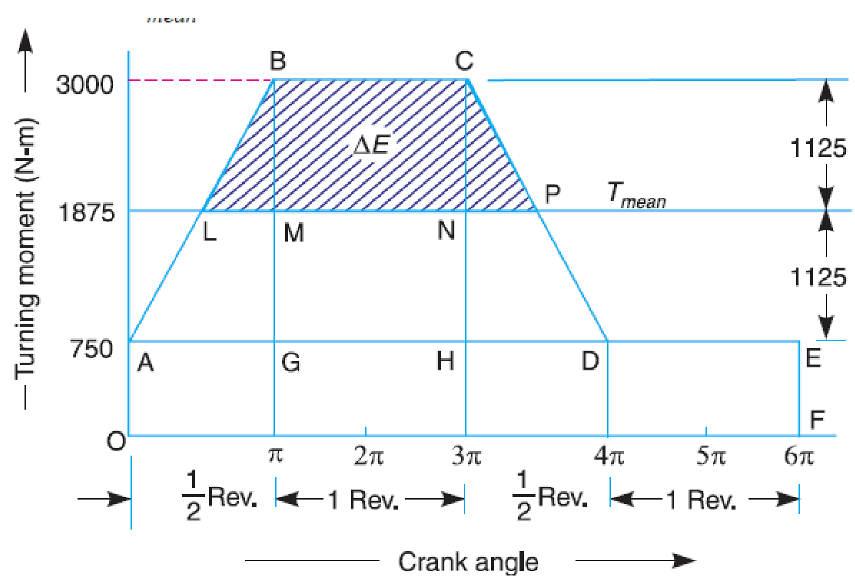
$$29.6m = 5404$$

$$m = 183kg$$

A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter. Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Given : N = 250 r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s ; m = 500 kg ; k = 600 mm = 0.6 m

Solution



The torque required for one complete cycle = Area of figure OABCDEF

= Area OAEF + Area ABG + Area BCHG + Area CDH

= (OF × OA) + (1/2 × AG × BG) + (GH × CH) + (1/2 × HD × CH)

= (6π × 750) + (1/2 × π × 2250) + (2π × 2250) + (1/2 × π × 2250)

= 11250 π N-m.

The torque required for one complete cycle = $T_{mean} \times 6\pi$

T_{mean} = 1875 N-m.

P = $T_{mean} \times \omega$ = 1875 × 26.2 = 49 125 W = 49.125 kW

From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BD}$$

$$\frac{LM}{\pi} = \frac{1125}{2250} = 0.5$$

$$LM = 0.5\pi$$

from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH}$$

$$\frac{NP}{\pi} = \frac{1125}{2250} = 0.5$$

$$NP = 0.5\pi$$

ΔE = Area LBCP = Area LBM + Area MBCN + Area PNC

$$= \frac{1}{2} LM \times BM + MN \times BM + \frac{1}{2} NP \times CN$$

$$= \frac{1}{2} \times 0.5\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\pi \times 1125$$

$$= 8837\ N - m$$

$\Delta E = mk^2\omega^2 C_s = 500.(0.6)^2(26.2)^2 C_s$

$123.559\ C_s = 86$

$C_s = 0.071 = 7.1\%$

A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2 per cent of the mean speed and the turning moment diagram during compression and expansion is assumed to be triangular in shape, find the moment of inertia of the flywheel.

Given

P = 20 kW = 20 × 103 W; C_s = ±2% = 4% = 0.04, N = 300 r.p.m.

ω = 2π × 300/60 = 31.42 rad/s,

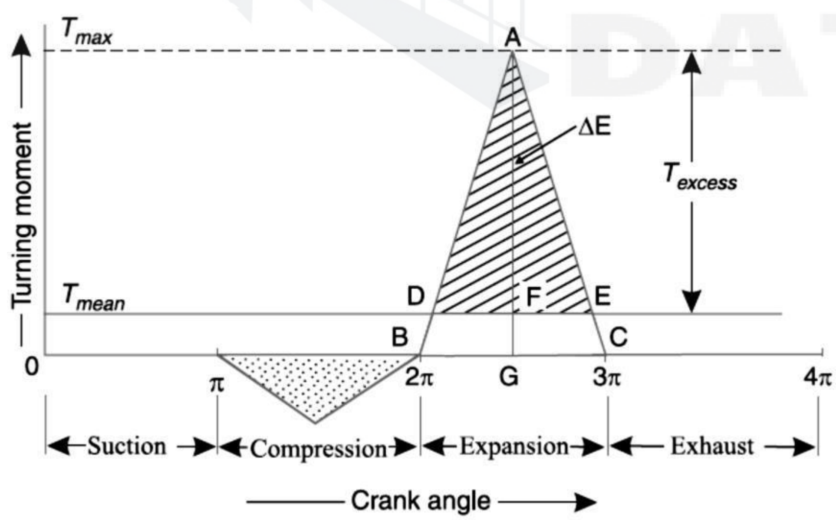
Solution

Number of working strokes per cycle,n = N/2 = 300 / 2 = 150

∴ Work done/cycle =P × 60/n = 20 × 103 × 60/150 = 8000 N-m

Net work done per cycle (during compression and expansion strokes)

$$= W_E - W_C = W_E - \frac{W_E}{3} = \frac{2}{3} W_E$$



work done during expansion stroke, W_E = 8000 × 3/2 = 12 000 N-m

$12\,000 =$ Area of triangle ABC $= \frac{1}{2} BC \times AG = \frac{1}{2} \pi AG$

$$AG = T_{max} = 12000 \times \frac{2}{\pi} = 7638\ N - m$$

$$T_{mean} = FG = \frac{Workdone/cycle}{crank\ angle\ /cycle} = \frac{8000}{4\pi} = 637 N - m$$

$T_{excess} = AF = AG – FG = 7638 – 637 = 7001$ N-m

from similar triangles ADE and ABC,

$$\frac{DF}{BC} = \frac{AF}{AG}$$

$$DE = \frac{AF}{AG} BC = \frac{7001}{7638} \pi = 2.88\ rad$$

$$\Delta E = Area\ of\ \Delta ADE = \frac{1}{2} DE \times AF = \frac{1}{2} \times 2.88 \times 7001 = 10081\ N - m$$

$$\Delta E = I\omega^2 C_s = (31.42)^2 0.04$$

$$39.5 I = 10081$$

$$I = 255.2\ kg - m^2$$

The turning moment curve for an engine is represented by the equation, T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find: 1. Power developed by the engine ; 2. Moment of inertia of flywheel in kg-m2, if the total fluctuation of speed is not exceed 1% of mean speed which is 180 r.p.m; and 3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.

Given : T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m ; N = 180 r.p.m. or

$\omega = 2\pi \times 180/60 = 18.85$ rad/s, $C_s = 1\% = 0.01$

Solution

Work done per revolution

$$= \int_0^{2\pi} T d\theta = \int_0^{2\pi} (20000 + 9500 Sin 2\theta - 5700 cos 2\theta)$$

$$= \left[20000\theta - \frac{9500 cos 2\theta}{2} - \frac{5700 sin 2\theta}{2}\right]_0^{2\pi}$$

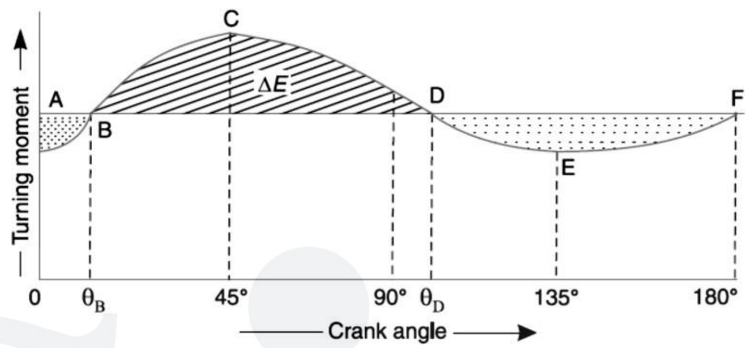
$$= 20000 \times 2\pi = 40000\pi\ N - m$$

$$T_{mean} = \frac{Workdone}{2\pi} = \frac{40000}{2\pi} = 20000N - m$$

1. Power developed by the engine

$$= T_{mean}\omega = 20000 \times 18.85 = 377000W = 377KW$$

2. Moment of inertia of the flywheel



$T = T_{mean}$

$$20000 + 9500Sin2\theta - 5700cos2\theta = 20000$$

$2\theta = 31°$ or $\theta = 15.5°$

$\theta_B = 15.5°$ and $\theta_D = 90° + 15.5° = 105.5°$

Maximum fluctuation of energy

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean})d\theta$$

$$= \int_{15.5°}^{105.5°} (20000 + 9500Sin2\theta - 5700cos2\theta)$$

$$= \left[-\frac{9500cos2\theta}{2} - \frac{5700sin2\theta}{2}\right]_{15.5°}^{105.5°}$$

$$= 11078\ N - m$$

$$\Delta E = I\omega^2 C_s = (18.85)^2 0.01$$

$$3.55I = 11078$$

$$I = 3121\ kg - m^2$$

3. Angular acceleration of the flywheel

$T_{excess} = T - T_{mean}$

$= 20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20000$

$= 9500 \sin 2\theta - 5700 \cos 2\theta$

$= 9500 \sin 90° - 5700 \cos 90° = 9500$ N-m

Torque = $I.\alpha = 3121 \times \alpha$

$\alpha = 9500/3121 = 3.044$ rad /s^2

Civil DATAS

UNIT II BALANCING

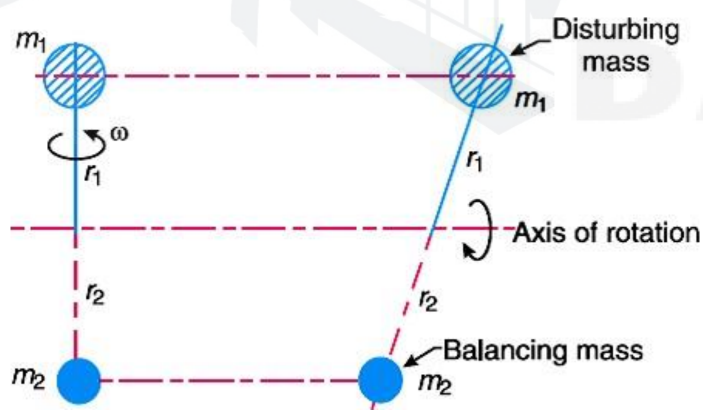
Introduction

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimize pressure on the main bearings when an engine is running.

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

1. Balancing of a single rotating mass by a single mass rotating in the same plane.



$$F_1 = m_1\omega^2 r_1 = F_2 = m_2\omega^2 r_2$$

Consider any number of masses (say four) of magnitude $m1$, $m2$, $m3$ and $m4$ at distances of $r1$, $r2$, $r3$ and $r4$ from the axis of the rotating shaft. Let θ1, θ2,θ3 and θ4 be the angles of these masses with the horizontal line *OX*, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through *O* and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s. The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussedbelow :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e*. Σ*H* and Σ*V* . We know that

Sum of horizontal components of the centrifugal forces,

$\Sigma H = m1 \cdot r1 \cos\theta 1 + m2 \cdot r2 \cos\theta 2 + \ldots\ldots$

and sum of vertical components of the centrifugal forces,

$\Sigma V = m1 \cdot r1 \sin \theta 1 + m2 \cdot r2 \sin \theta 2 + \ldots\ldots$

3. Magnitude of the resultant centrifugal force,

$$\boldsymbol{F_C} = \sqrt{(\boldsymbol{\Sigma H})^2 + (\boldsymbol{\Sigma V})^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan \theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction*.**

6. Now find out the magnitude of the balancing mass, such that

FC m r

where m = Balancing mass, and

r = Its radius of rotation.

2. Graphical method

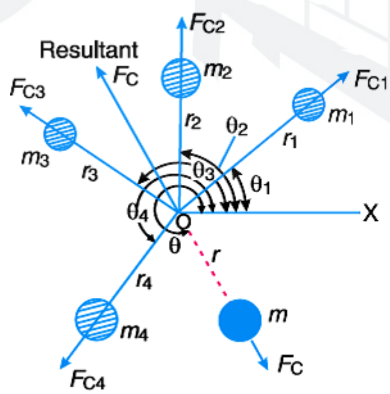
The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig.
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass *m*1 (or *m*1.*r*1) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de*to represent centrifugal forces of other masses *m*2, *m*3 and *m*4 (or *m*2.*r*2,*m*3.*r*3 and *m*4.*r*4).
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 21.4 (*b*).
5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass (*m*) at a given radius of rotation (*r*), such that

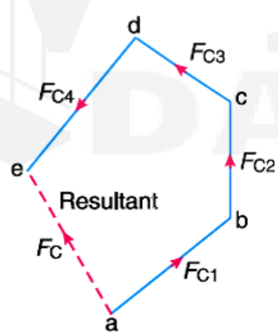
$m \cdot \omega 2 \cdot r$ = Resultant centrifugal force

or$m.r$= Resultant of *m*1.*r*1, *m*2.*r*2, *m*3.*r*3 and *m*4.*r*4

2. Balancing of different masses rotating in the same plane.



(*a*) Space diagram.



(*b*) Vector diagram.

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance,*i.e.*the resultant force must be zero.

2. The couples about the reference plane must balance,*i.e*. the resultant couple must be zero.

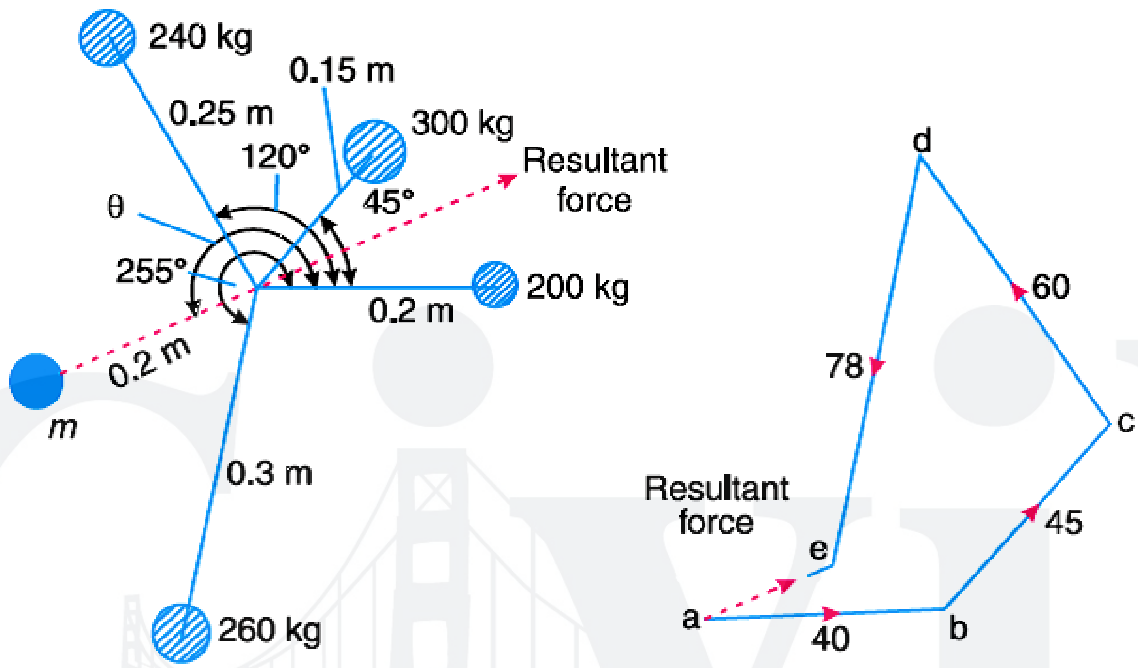
Let us now consider four masses *m*1, *m*2, *m*3 and *m*4 revolving in planes 1, 2, 3 and 4 respectively as shown infig.

The relative angular positions of these masses are shown in the end view. The magnitude of the balancing masses mL and mM in planes L and M may be obtained asdiscussed below :

1. Take one of the planes, say L as the reference plane (R.P.). The distances of all the otherplanes to the left of the reference plane may be regarded as negative, and those to theright as positive.

2. Tabulate the data as shown in Table The planes are tabulated in the same order inwhich they occur, reading from left to right.

Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kgrespectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectivelyand the angles between successive masses are 45°, 75° and 135°. Find the position and magnitudeof the balance mass required, if its radius of rotation is 0.2 m.



(*a*) Space diagram. (*b*) Vector diagram

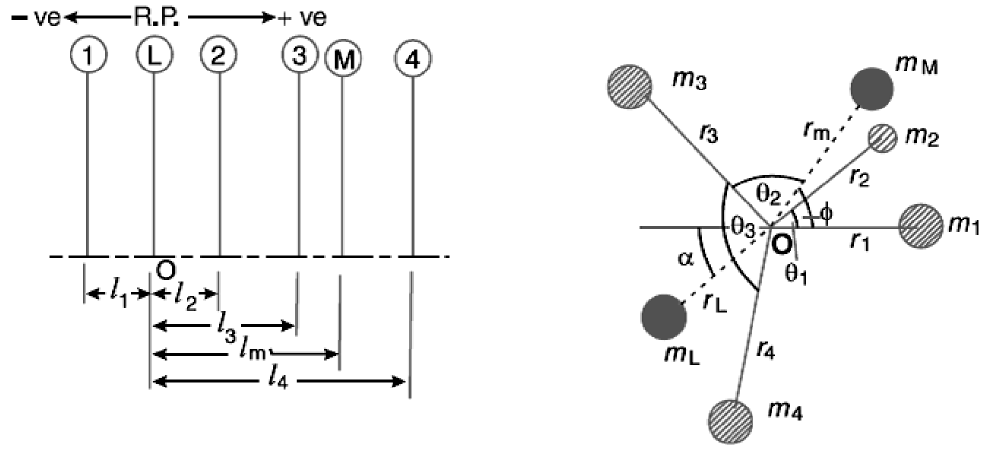
$m_1.r_1 = 200 \times 0.2 = 40$ kg-m $\quad m_3.r_3 = 240 \times 0.25 = 60$ kg-m

$m_2.r_2 = 300 \times 0.15 = 45$ kg-m $\quad m_4.r_4 = 260 \times 0.3 = 78$ kg-m

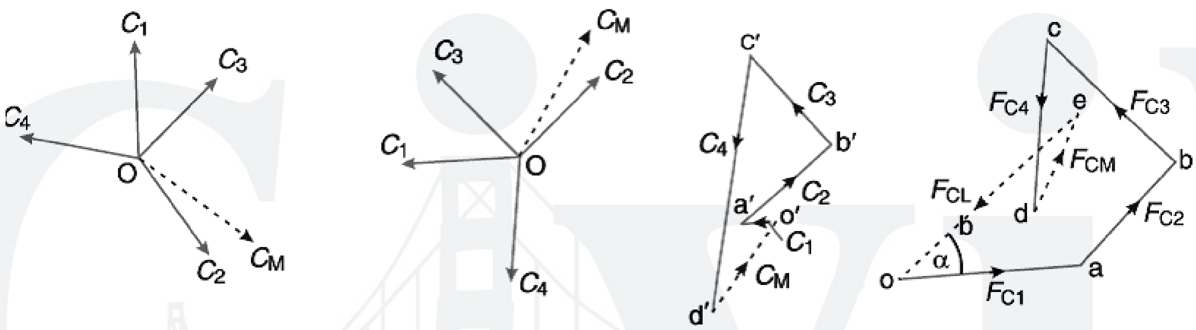
$m = 23/0.2 =$ **115 kg Ans.** $\quad \theta = 201°$ **Ans.**

3. Balancing of different masses rotating in different planes

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



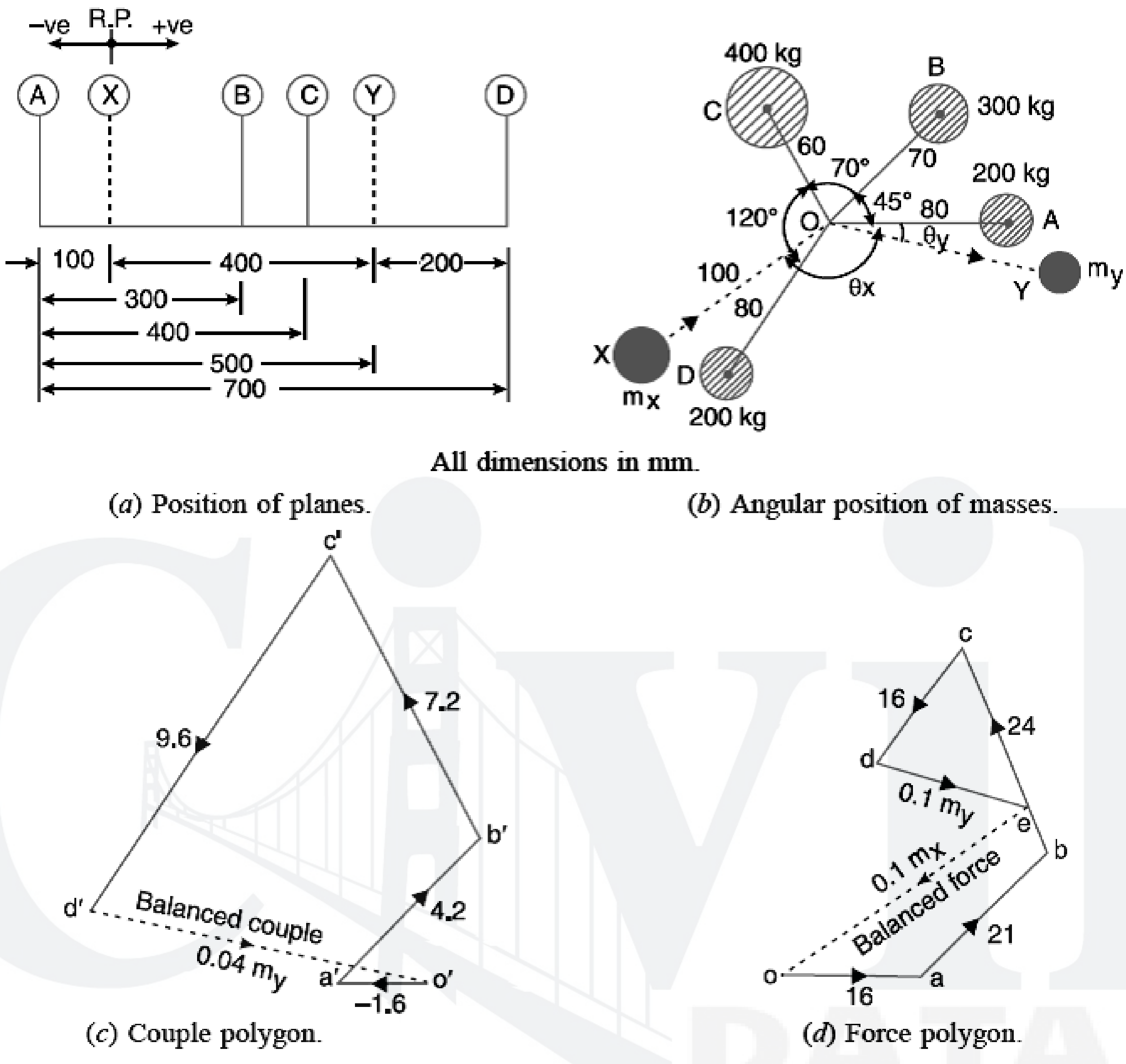
(*a*) Position of planes of the masses. (*b*) Angular position of the masses.



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg,400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planesmeasured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measuredanticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placedin planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, findtheir magnitudes and angular positions.

Plane (1)	*Mass (m)* *kg* (2)	*Radius (r)* *m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6



All dimensions in mm.

(*a*) Position of planes.

(*b*) Angular position of masses.

(*c*) Couple polygon.

(*d*) Force polygon.

$m_Y = 182.5$ kg **Ans.** $\theta_Y = 12°$ $m_X = 355$ kg **Ans.** $\theta_X = 145°$

Four masses A, B, C and D as shown below are to be completely balanced.

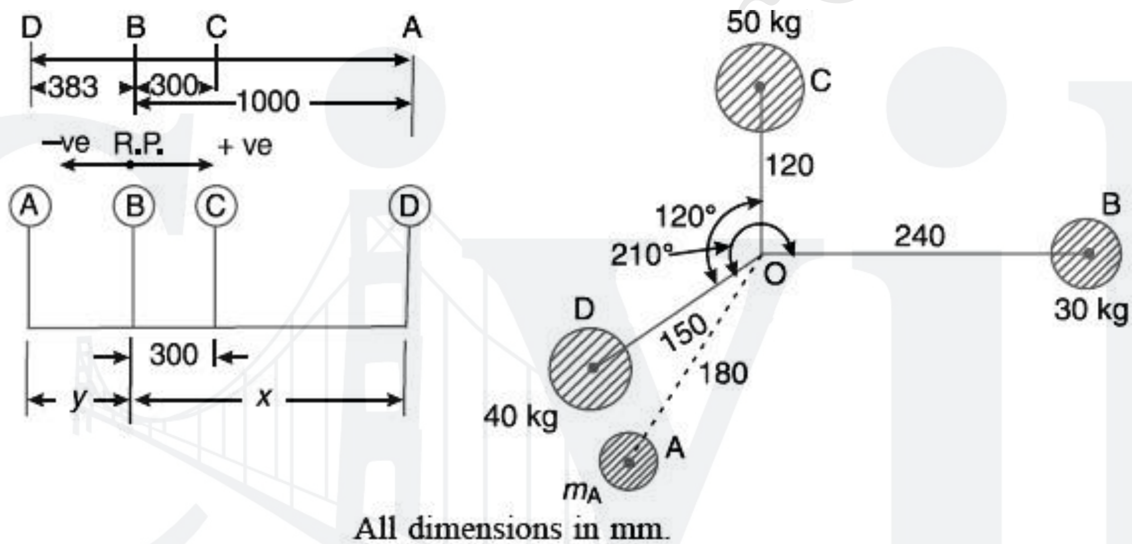
	A	B	C	D
Mass	**-**	**30**	**50**	**40**
Radius	**180**	**240**	**120**	**150**

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the samesense. Find :

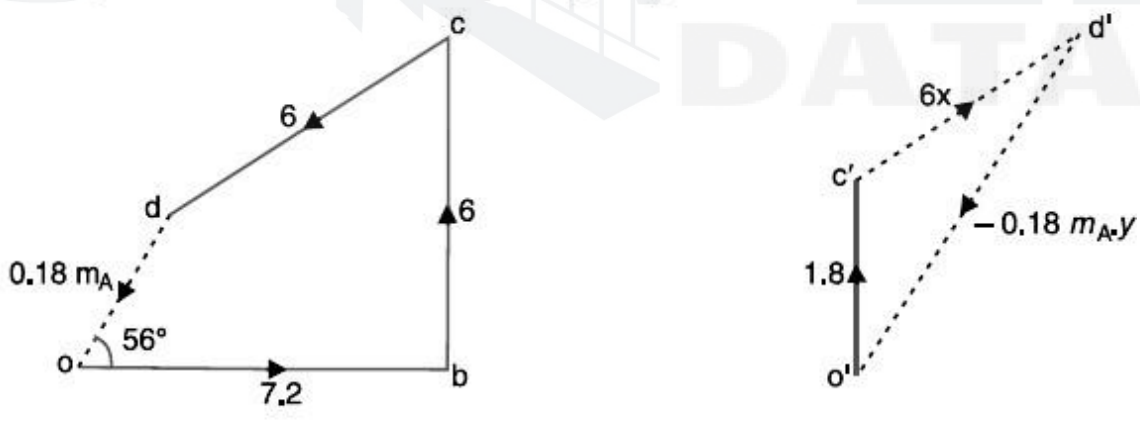
1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

Plane (1)	*Mass* *(m) kg* (2)	*Radius* *(r) m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	m_A	0.18	0.08 m_A	$-y$	$-0.18\ m_A y$
B (*R.P*)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$



(*a*) Position of planes. (*b*) Angular position of masses.



(*c*) Force polygon. (*d*) Couple polygon.

Balancing of Reciprocating Masses

Introduction

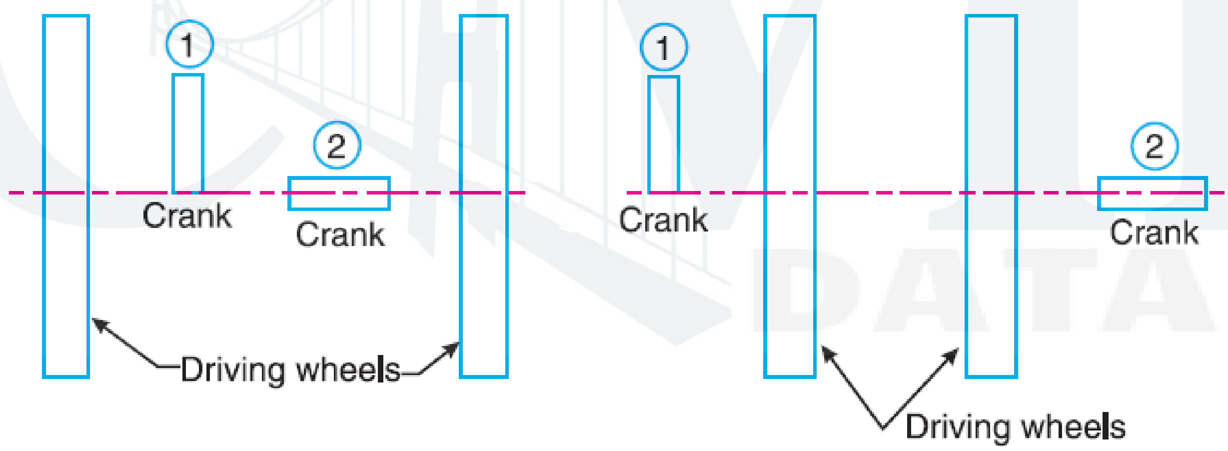
The various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as **unbalanced force** or **shaking force**.

Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives maybe classified as :

1. Inside cylinder locomotives ; and **2.** Outside cylinder locomotives.

In the **inside cylinder locomotives**, the two cylinders are placed in between the planes oftwo driving wheels as shown in Fig. ; whereas in the **outside cylinder locomotives**, the twocylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown inFig). The locomotives may be**(a)** Single or uncoupled locomotives ; and **(b)** Coupled locomotives



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

A **single** or **uncoupled locomotive** is one, in which the effort is transmitted to one pair of the wheels only ; whereas in **coupled locomotives,** the driving wheels are connected to the leading and trailing wheel by an outside coupling rod.

Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives

We have discussed in the previous article that the reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force alongthe line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke; and **2.** Swaying couple.

The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a **hammer blow**.

Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is knownas **tractive force**

Swaying Couple

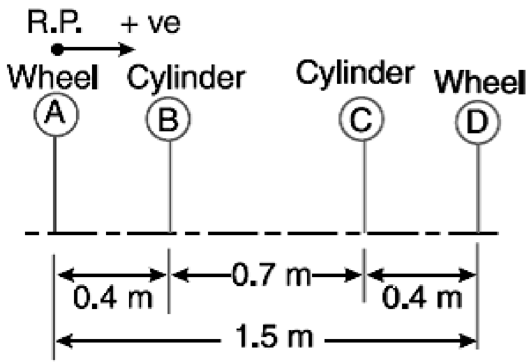
This couple has swaying effect about a vertical axis, and tends to sway the engine alternatelyin clockwise and anticlockwise directions. Hence the couple is known as **swaying couple.**

Hammer Blow

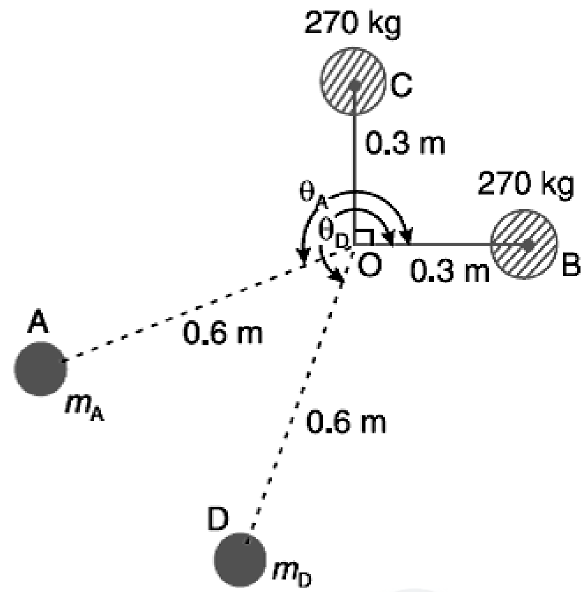
We have already discussed that the maximum magnitude of the unbalanced force along theperpendicular to the line of stroke is known as **hammer blow**.

An inside cylinder locomotive has its cylinder centre lines 0.7 m apart andhas a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin,and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. Thecranks are at right angles.The whole of the rotating and 2/3 of the reciprocatingmasses are to be balanced by masses placed at aradius of 0.6 m. Find the magnitude and direction of thebalancing masses.Find the fluctuation in rail pressure under onewheel, variation of tractive effort and the magnitude ofswaying couple at a crank speed of 300 r.p.m.

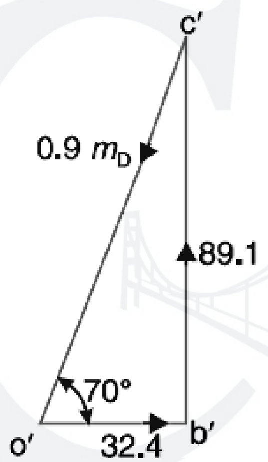
Plane (1)	*mass.* *(m) kg* (2)	*Radius* *(r)m* (3)	*Cent. force* ÷ ω^2 *(m.r) kg-m* (4)	*Distance from plane A (l)m* (5)	*Couple* ÷ ω^2 *(m.r.l) kg-m*^{2} (6)
A (*R.P.*)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6m_D$	1.5	0.9 m_D



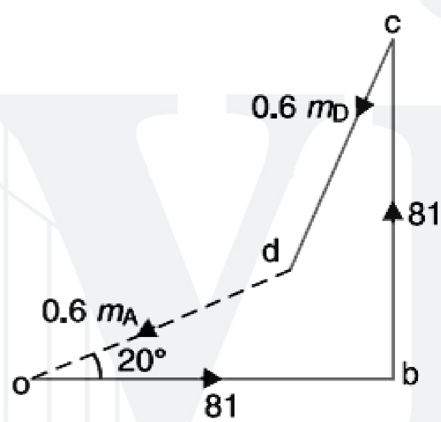
(*a*) Position of planes.



(*b*) Angular position of masses.



(*c*) Couple polygon.



(*d*) Force polygon.

$m_D = 105$ kg $\theta_D = 250°$ $m_A = 105$ kg $\theta_A = 200°$

Fluctuation in rail pressure

We know that each balancing mass

$= 105$ kg

$\therefore$ Balancing mass for rotating masses,

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

$\therefore$ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6\,(31.42)^2\,0.6 = 27\,602 \text{ N. } \textbf{Ans.} \qquad \ldots (\because\ b = r_A = r_D)$$

Civil DATAS

Variation of tractive effort

We know that maximum variation of tractive effort

$$= \pm\sqrt{2}(1-c)m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2 0.3\,\text{N}$$

$= \pm$ 25 127 N **Ans.** ... ($\because$ $r = r_B = r_C$)

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3\ \text{N-m}$$

= 8797 **N-m Ans.**

Civil DATAS

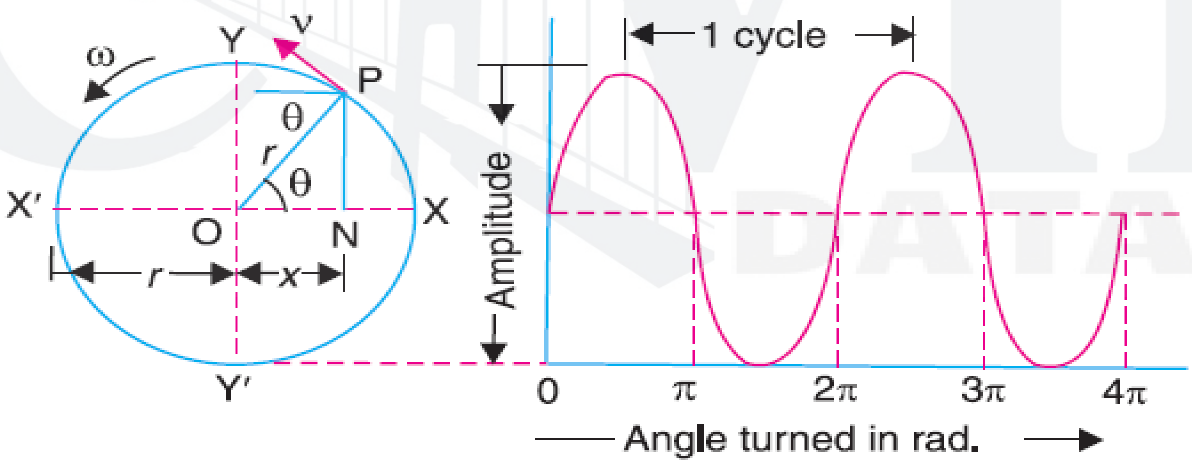
UNIT III SINGLE DEGREE FREE VIBRATION

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration– Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems

Terms Used in Vibratory Motion

1. ***Period of vibration or time period.*** It is the time interval after which the motionis repeated itself. The period of vibration is usually expressed in seconds.
2. ***Cycle.*** It is the motion completed during one time period.
3. ***Frequency.*** It is the number of cycles described in one second. In S.I. units, the frequencyis expressed in hertz (briefly written as Hz) which is equal to one cycle per second.
4. **Amplitude (X).**The maximum displacement of a vibrating body from the mean position.

Differential Equation of SHM



$$x = X\, sin\omega t$$

$$v = \frac{dx}{dt} = \omega X cos\omega t$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 X sin\omega t = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 = 0$$

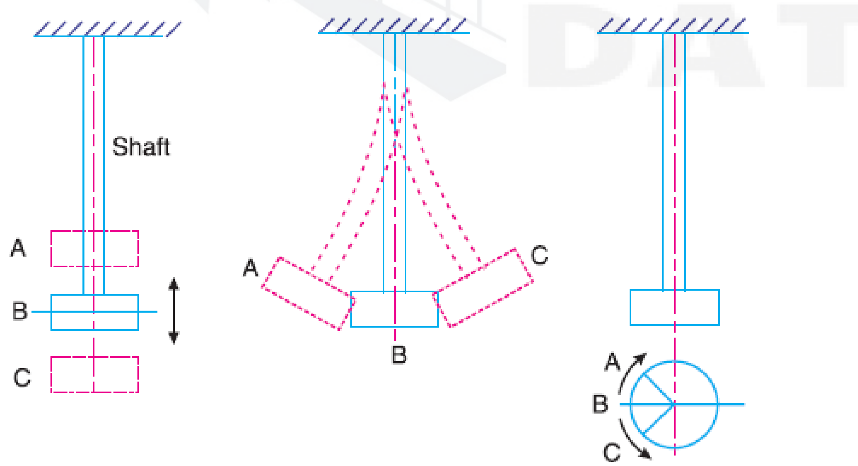
$$Time\ Period, t_p = \frac{2\pi}{\omega}$$

$$Frequency, f = \frac{1}{t_p} = \frac{\omega}{2\pi}$$

Types of Vibratory Motion

1. ***Free or natural vibrations*.** When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations*.** The frequency of the free vibrations is called ***free or natural frequency*.**
2. ***Forced vibrations*.** When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations*.** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.
3. ***Damped vibrations*.** When there is a reduction in amplitude over every cycle of vibration, the motion is said to be ***damped vibration*.** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

Types of Free Vibrations



B = Mean position ; *A* and *C* = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

1. ***Longitudinal vibrations*.** When the particles of the shaft or disc moves parallel to theaxis of the shaft, as shown in Fig. 23.1 (*a*), then the vibrations are known as ***longitudinal vibrations*.** In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

2. ***Transverse vibrations*.** When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Fig. 23.1 (*b*), then the vibrations are known as ***transverse vibrations*.** In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.

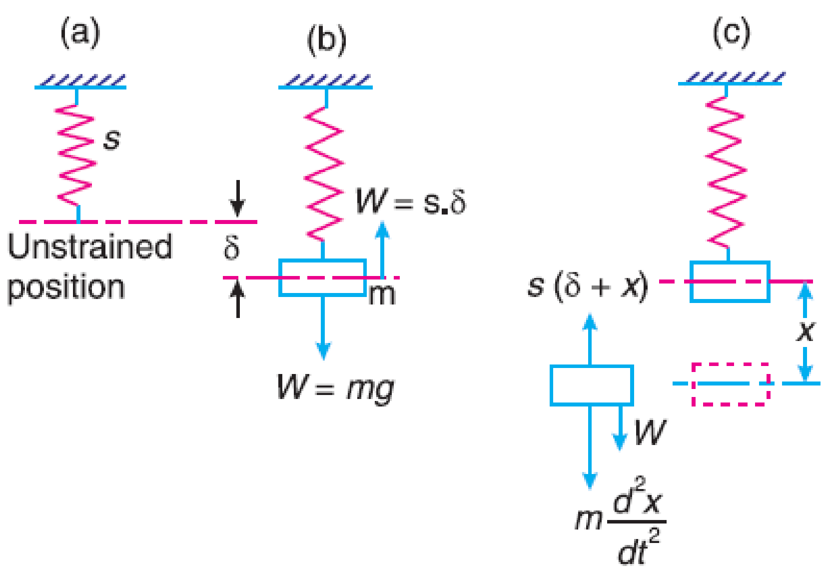
3. ***Torsional vibrations***. When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Fig. 23.1 (*c*), then the vibrations are known as ***torsional vibrations*.** In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.

UNDAMPED FREE LONGITUDINAL VIBRATIONS

There are three methods

1. Equlibrium Method or Newtlon's Method
2. Energy Method
3. Rayleigh's Method

Equilibrium Method (or Newton's Method)



$$s\delta = mg$$

Acceleration force = Mass x Acceleration

$$= m.\frac{d^2y}{dt^2} \text{ (downwards)}$$

Inertia force = - Acceleration force

$$\text{Spring force} = W - s(\delta + x) = W - s\delta - sx$$

$$= mg - s\delta - sx$$

$$= sx$$

$$\text{Inertia force} + \sum \text{External force} = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0$$

Natural Frequency equation is

$$\frac{d^2x}{dt^2} + \omega_n^2 x = 0$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

Where natural frequency

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{0.4985}{\sqrt{\delta}}$$

Energy Method

The sum of Kinetic and the Potential energies, constant

$$K.E + P.E = \text{Constant}$$

$$\frac{d}{dt}(K.E + P.E) = 0$$

We know that,

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{1}{2}\left(\frac{dx}{dt}\right)^2$$

and P.E = Mean force x Displacement

$$= \left(\frac{Force\ at\ A + Force\ at\ B}{2}\right) x\ Displacement$$

$$= \left(\frac{0 + sx}{2}\right) x = \frac{1}{2} sx^2$$

Substituting P.E and K.E equation,

$$\frac{d}{dt}\left(\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} sx^2\right) = 0$$

$$m\frac{d^2x}{dt^2} + sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} = 0$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

Rayleigh's Method

$$\text{K.E at mean position} = \text{P.E at mean position}$$

$$x = X sin\omega_n t$$

$$\frac{dx}{dt} = \omega_n X \cos \omega_n t$$

$$v_{max} = \left(\frac{dx}{dt}\right)_{max} = \omega_n X$$

$$\text{K.E at mean position} = \text{P.E at mean position}$$

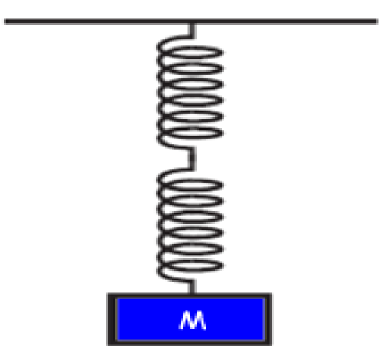
$$\frac{1}{2} m v_{max}^2 = \text{Mean force x Displacement}$$

$$\frac{1}{2}(\omega_n X)^2 = \left[\frac{0 + sX}{2}\right] X$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

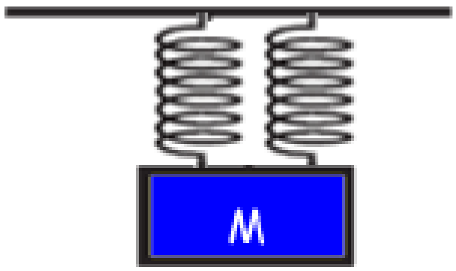
DETERMINATION OF EQUIVALENT SPRING STIFNESS

Spring in Series



$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2}$$

Spring in Parallel



$$S_{eq} = S_1 + S_2$$

DAMPED FREE LONGOTUDINAL VIBRATIONS

DAMPING

The damping can be defined as the resisting offered by a body to mation of a vibratory system.

DAMPING COEFFICIENT

The damping force per unit velocity is known as damping coefficient.

$$c = \frac{Damping\ force}{Velocity}$$

$$Damping\ force = c\ \frac{dx}{dt}$$

Dampers in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Dampers in Parallel

$$C_{eq} = C_1 + C_2$$

FREQUENCY OF FREE DAMED VIBRATIONS

$$\text{Acceleration force} = m\frac{d^2x}{dt^2}\ \text{(downwards)}$$

$$\text{Damping force} = c\frac{dx}{dt} \text{ (upwards)}$$

$$\text{Spring force} = \text{(upwards)}$$

$$m\frac{d^2x}{dt^2} + \left[c\frac{dx}{dt} + sx\right] = 0$$

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{s}{m}x = 0$$

The solution is

$$x = C_1e^{k_1t} + {}_2e^{k_2t}$$

$$k_1 = k_2 = \frac{-c}{2m} \pm \sqrt{\left(\frac{C}{2m}\right)^2 - \left(\frac{s}{m}\right)}$$

$$k_1 = k_{2= (-\zeta \pm\sqrt{\zeta^2-1})}$$

Critical Damping Coefficient and Damping Ratio

The ratio of $\left(\frac{c}{2m}\right)^2$ to $\left(\frac{s}{m}\right)$ represents the degree of dampness of provided in the system and its square root is known as damping factor or damping ratio.

$$\sqrt{\left(\frac{C}{2m}\right)^2 - \left(\frac{s}{m}\right)} = 0$$

$$\zeta = \sqrt{\frac{\left(\frac{C}{2m}\right)^2}{\frac{s}{m}}} == \frac{c}{2\sqrt{sm}}$$

Damping coefficient $c = 2\zeta\sqrt{sm} = 2\zeta m\omega_n = 2\zeta\frac{s}{\omega_n}$

Critical damping coefficient is $\zeta = 1$

$$\zeta = \frac{c}{c_c}$$

$$c_c = 2m\omega_n$$

When

$\zeta = 1$, the damper is critical, the equation

$$x = (A + Bt)e^{-\omega_n t}$$

$\zeta > 1$, the damper is over damped, the equation

$$k_1 = k_{2= (-\zeta \pm\sqrt{\zeta^2-1})}$$

$$x = C_1 e^{k_1 t} + {}_2 e^{k_2 t}$$

$\zeta < 1$, the damper is under damped, the equation

$$x = Xe^{-\zeta\omega_n t} sin(\omega_d t + \phi)$$

$$\omega_d = (\sqrt{1 - \zeta^2})$$

LOGARITHMIC DECREMENT

The ratio of two successive oscillations is constant in an underdamped system. Natural logarithm of this ratio is called logarithmic decrement and denoted by δ

$$\delta = \ln\left(\frac{X_n}{X_{n+1}}\right) = \ln e^{(\zeta\omega_n t_d)} = \zeta\omega_n t_d = \frac{1}{n} ln\left(\frac{X_o}{X_n}\right)$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \ (or)\ \zeta^2 = \frac{\delta^2}{4\pi^2 + \delta^2}$$

A machine weights 20kg and is supported on springs and dashpots. The total stiffness of the springs is 12N/mm and the damping is 0.2N/mm/s. the system is initially at rest and a velocity of 125mm/s is imparted to the mass. Determine the (i) Displacement and Velocity of mass as a function of time (ii) Displacement and Velocity after 0.5s.

Solution

$$c_c = 2m\omega_n$$

$$\omega_n = \sqrt{\frac{s}{m}} = 24.49\ rad/s$$

$$c_c = 2m\omega_n = 979.795\ N/m/s$$

$$\zeta = \frac{c}{c_c} = 0.204 < 1\ (Under\ damped)$$

$$x = Xe^{-\zeta\omega_n t} sin(\omega_d t + \phi)$$

$$\omega_d = (\sqrt{1-\zeta^2}) = (\sqrt{1 - 0.204^2})\ 24.29 = 23.97\ rad/s$$

$$x = Xe^{-4.99t} s(23.97t + \phi)$$

At x = 0, t = 0,

$$0 = Xe^0 s(0 + \phi)$$

$$sin\varphi = 0, i.e., \varphi = 0$$

At t = 0, $v = \frac{}{dt} = 0.125\ m/s$

$$x = Xe^{-4.99t}s(23.97t)$$

$$\frac{dx}{dt} = X\ e^{-4.99}[23.97cos23.97t] + X(-4.99e^{-4.99t})[sin23.97t]$$

$$\frac{dx}{dt} = 23.97X$$

$$X = 5.214 \times 10^{-3}\ m$$

$$\boldsymbol{x = 5.214 \times 10^{-3}e^{-4.99t}sin(23.97t + \phi)}$$

$$\boldsymbol{\frac{dx}{dt} = e^{-4.99t}[0.124(cos23.97t) - 0.02608(sin23.97t)]}$$

Case (ii) t = 0.5s

$$x = 5.214 \times 10^{-3}e^{-4.99t}s(23.97t + \phi)$$

$$x = 5.214 \times 10^{-3}e^{-4.99(0.5)}sin\ (23.97(0.5)\frac{180}{\pi})$$

$$x = 8.932 \times 10^{-5}\ m$$

$$\frac{dx}{dt} = e^{-4.99}[0.124(cos23.97t) - 0.02608(sin23.97t)]$$

$$\frac{dx}{dt} = e^{-4.99(0.5)}\ [0.124\ (cos23.97(0.5)\ \frac{180}{\pi}) - 0.02608(sin23.97(0.5)\frac{180}{\pi})]$$

$$v = \frac{dx}{dt} = 9.479 \times 10^{-3}\ m/s$$

In a Single degree damped vibrating system; a suspended mass of 3.75kg makes 12 Oscillations in 7 seconds whendisturbed from its equilibrium position. The amplitude of vibrations reduce to 0.33 of its initial value after four oscillations. Determine (i) stiffness of the spring (ii) logarithmic decrement (iii) damping factor and (iv) the damping co-efficient

Given

m = 3.75kg, $N_T = 12$, $t_T = 7S$, $X_4 = 0.33\ X_0$, n=4

Solution

$$f_n = \frac{N_T}{t_T} = \frac{12}{7} = 1.714HZ$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{S}{m}}$$

$$S = 434.923\ N/m$$

$$\delta = \frac{1}{n}\ln\left(\frac{X_0}{X_n}\right) = 0.277$$

$$\delta = \frac{2\pi\zeta}{\sqrt{(1-\zeta^2)}} = 0.044$$

$$\zeta = \frac{C}{C_c}$$

$$\omega_n = \sqrt{\frac{S}{m}} = 10.769\ rad/s$$

$$C = 3.553\ N/m/s$$

A shaft 100 mm diameter and 1 metre long has one of its end fixed and the other end carries a disc of mass 500kg. The modulus of elasticity for the shaft material is 200 GN/m^2. Determine the frequency of longitudinal vibrations.

Given

d = 100mm, l = 1m, m = 500kg, E = 200GN/m^2

Solution

$$\delta = \frac{Wl}{AE} = 3.122 \times 10^{-6}\ m$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = 1772.629\ rad/s$$

$$f_n = \frac{\omega_n}{2\pi} = 282.122\ Hz$$

A machine weighs 18 kg and is supported on springs anddashpots. The total stiffness of the springs is 12 *N/mm* and damping is *0.2 N/mm/s* the system is initially at rest

and a velocity of 120 *mm/s* imparted to the mass. Determine (1) the displacement and velocity of mass as a function of time (2) the displacement and Velocity after O.4s.
Solution

$$c_c = 2m\omega_n$$

$$\omega_n = \sqrt{\frac{s}{m}} = 25.819\ rad/s$$

$$c_c = 2m\omega_n = 929.576\ N/m/s$$

$$\zeta = \frac{c}{c_c} = 0.215 < 1\ (Under\ damped)$$

$$x = Xe^{-\zeta\omega_n t}sin(\omega_d t + \phi)$$

$$\omega_d = (\sqrt{1-\zeta^2}) = (\sqrt{1-0.215^2})\,25.819 = 25.215\ rad/s$$

$$x = Xe^{-5.55t}s(25.215t + \phi)$$

At x = 0, t = 0,

$$0 = Xe^0 s(0 + \phi)$$

$$sin\varphi = 0, i.e., \varphi = 0$$

At t = 0, $v = \frac{dx}{dt} = 0.125\ m/s$

$$x = Xe^{-5.55t}s(25.215t)$$

$$\frac{dx}{dt} = X\ e^{-5.55}[25.215cos25.215t] + X(-5.55e^{-5.55t})[sin25.215t]$$

$$\frac{dx}{dt} = 25.215X$$

$$X = 4.759 \times 10^{-3}\ m$$

$$\boldsymbol{x = 4.759 \times 10^{-3}e^{-5.55t}sin(25.215t + \phi)}$$

$$\boldsymbol{\frac{dx}{dt} = e^{-5.55t}[0.119(cos25.215t) - 0.0264(sin25.215t)]}$$

Case (ii) t = 0.4s

$$x = 4.759 \times 10^{-3}e^{-5.55t}s(25.215t + \phi)$$

$$x = 4.759 \times 10^{-3} e^{-5.55(0.4)} sin\,(25.215(0.4)\,\frac{180}{\pi})$$

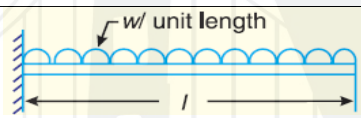
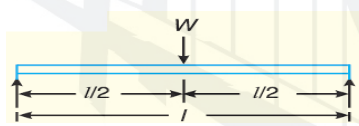
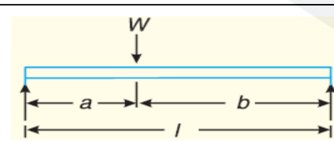
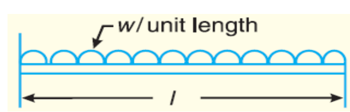
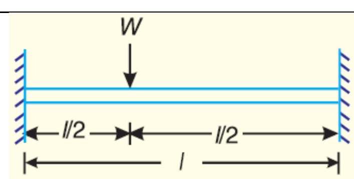
$$x = -3.173 \times 10^{-4}\ m$$

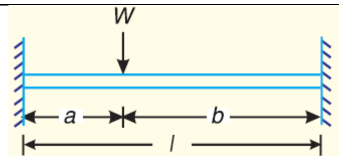
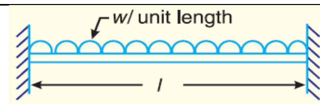
$$\frac{dx}{dt} = e^{-5.55}[0.119(cos25.215t) - 0.0264(sin25.215t)]$$

$$\frac{dx}{dt} = e^{-5.55(0.4)}\,[0.119\,(cos25.215(0.4)\,\frac{180}{\pi}) - 0.0264(sin25.215(0.4)\,\frac{180}{\pi})]$$

$$v = \frac{dx}{dt} = -8.437 \times 10^{-3}\ m/s$$

TRANSVERSE VIBRATIONS

SI.NO	TYPES OF BEAM	DIAGRAM	DEFLECTION	FREQUENCY
1	Cantilever beam Point load		$\delta = \frac{Wl^3}{3EI}$	$f_n = \frac{0.4985}{\sqrt{\delta}}$
2	Cantilever beam UDL		$\delta = \frac{Wl^4}{8EI}$	$f_n = \frac{0.621}{\sqrt{\delta}}$
3	Simply supported beam Point load		$\delta = \frac{Wl^3}{48EI}$	$f_n = \frac{0.4985}{\sqrt{\delta}}$
4	Simply supported beam Eccentric load		$\delta = \frac{Wa^2b^2}{3EIl}$	$f_n = \frac{0.4985}{\sqrt{\delta}}$
5	Simply supported beam UDL		$\delta = \frac{5wl^4}{384EI}$	$f_n = \frac{0.5615}{\sqrt{\delta}}$
6	Fixed beam Point load		$\delta = \frac{Wl^3}{192EI}$	$f_n = \frac{0.4985}{\sqrt{\delta}}$

7	Fixed beam Eccentric load		$\delta = \frac{Wa^3b^3}{3EIl}$	$f_n = \frac{0.4985}{\sqrt{\delta}}$
8	Fixed beam UDL		$\delta = \frac{Wl^3}{384EI}$	$f_n = \frac{0.571}{\sqrt{\delta}}$

Dunkerey's Method

$$\frac{1}{f_n^2} = \frac{1}{f_{n1}^2} + \frac{1}{f_{n2}^2} + \frac{1}{f_{n3}^2} + \frac{1}{f_{n4}^2} + \cdots$$

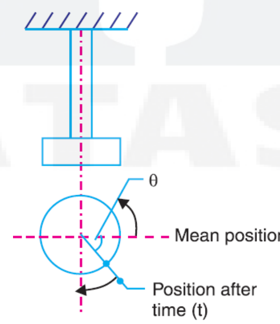
$$\omega_{cr} = \omega_n = \sqrt{\frac{s}{m}}$$

$$y = \frac{e}{\left(\frac{N_{cr}}{N}\right)^2 - 1}$$

$$N_{cr}\frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \text{X } 60 = f_n \text{x } 60$$

Torsional Vibrations

Single rotor



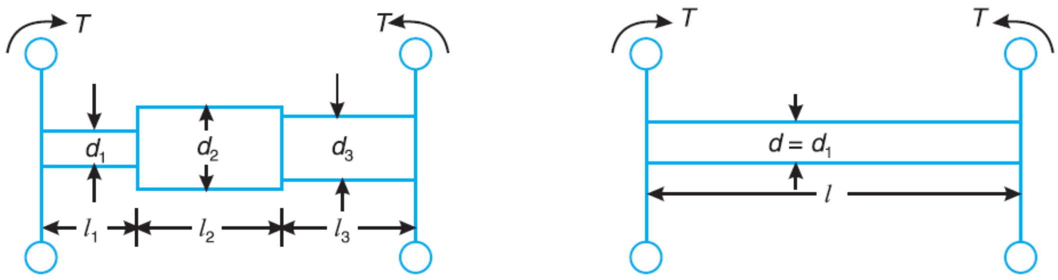
$$\omega = \sqrt{\frac{q}{I}}$$

$$t_p = \frac{2\pi}{\omega}$$

$$f_n = \frac{1}{t_p}$$

$$q = \frac{T}{\theta} = \frac{CJ}{l}$$

Torsionally Equivalent shaft



(*a*) Shaft of varying diameters. (*b*) Torsionally equivalent shaft.

$$\theta = \theta_1 + \theta_2 + \theta_3 + \cdots$$

$$\frac{l_e}{{d_e}^4} = \frac{l_1}{{d_{e1}}^4} + \frac{l_2}{{d_{e2}}^4} + \frac{l_3}{{d_{e3}}^4} + \cdots$$

$$_e = \left(\frac{d_e}{d_1}\right)^4 + l_2 \left(\frac{d_e}{d_2}\right)^4 + l_3 \left(\frac{d_e}{d_3}\right)^4 + \cdots$$

$d_e = d_1$

$$l_e = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4 + \cdots$$

The shaft carries two masses. The mass A is 300 kg with radius of gyration of 0.75m and the mass B is 500 kg with radius of gyration of 0.9m. the shaft is a stepped shaft having, 100 mm diameter for 300 mm length, 150 mm diameter for 160 mm length, 120 mm diameter for 125 mm length and 90mm diameter for 400mm length. Determine the frequency of the natural torsional vibration. It is desired to have node at the mid-section of shaft of 120mm diameter by changing the diameter of the section having 90mm diameter, what will be the new diameter. Take G = 84 GN/m²

Solution

$$l_e = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4 + l_4 \left(\frac{d_1}{d_4}\right)^4$$

$$l_e = 1.001\ m$$

$$I_A = m_A k_A^2 = 300 \times 0.75^2 = 168.75\ m^2$$

$$I_B = m_B k_B^2 = 500 \times 0.9^2 = 405\ m^2$$

$$J = \frac{\pi}{32} d_1^4 = 9.817 \times 10^{-6}\ m^4$$

$$l_A I_A = l_B I_B$$

$$l_A = l_B \frac{I_B}{I_A} = 2.4 l_B$$

$$l_e = l_A + l_B$$

$$l_B = 0.288\ m$$

$$l_A = 0.674\ m$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\mathcal{G}}{l_A I_A}} = 13.55\ Hz$$

Case (ii)

$$l_A = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + \frac{l_3}{2}\left(\frac{d_1}{d_3}\right)^4 = 0.3617\ m$$

$$l_A I_A = l_B I_B$$

$$l_B = 0.1507\ m$$

$$l_A = 0.3617\ m$$

$$l_B = \frac{l_3}{2}\left(\frac{d_1}{d_3}\right)^4 + l_4\left(\frac{d_1}{d_4}\right)^4 = 0.134 m$$

A steel shaft ABCD 1.5m long has flywheel at its ends A and D. The mass of the flywheel A is 600 kg and has a radius of gyration of 0.6m. The mass of the flywheel D is 800 kg and has a radius of gyration of 0.9m. The connecting shaft has a diameter of 50mm for the portion AB which is 0.4m long, and has a diameter of 50mm for the portion of BC which is 0.5m long and has a diameter of D mm for the portion CD which is 0.6m long. Determine (i) the dia D of the portion CD so that the node of the torsional vibration of the system will be at the centre of the length BC and (ii) the natural frequency of the torsional vibrations. The modulus of rigidity for the shaft material is 80GN/m².

Solution

$$\mathrm{I_A} = m_A k_A^2 = 600 \times 0.6^2 = 216\ m^2$$

$$\mathrm{I_B} = m_B k_B^2 = 800 \times 0.9^2 = 648\ m^2$$

$$l_A I_A = l_B I_B$$

$$l_A = l_B \ \frac{I_B}{I_A} = 3l_B$$

$$l_A = 3l_D$$

$$J = \frac{\pi}{32} d_1{}^4 = 6.135 \times 10^{-7}\, m^4$$

$$l_A = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4 + l_4 \left(\frac{d_1}{d_4}\right)^4$$

$$l_e = l_A + l_B$$

$$l_B = 0.288\, m$$

$$l_A = 0.674\, m$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{l_A I_A}} = 13.55\, Hz$$

Case (ii)

$$l_A = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + \frac{l_3}{2} \left(\frac{d_1}{d_3}\right)^4 = 0.3617\, m$$

$$l_A I_A = l_B I_B$$

$$l_B = 0.1507\, m$$

$$l_A = 0.3617\, m$$

$$l_B = \frac{l_3}{2} \left(\frac{d_1}{d_3}\right)^4 + l_4 \left(\frac{d_1}{d_4}\right)^4 = 0.134m$$

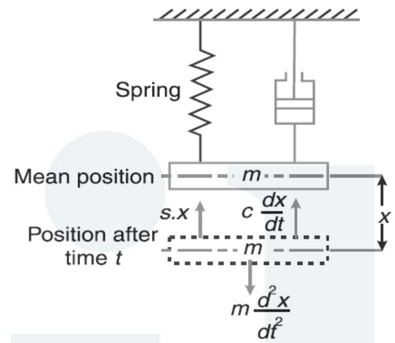
Civil DATAS

UNIT IV FORCED VIBRATION

Response of one degree freedom systems to periodic forcing – Harmonic disturbances – Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolationvibration measurement.

FORCED VIBRATIONS WITHCONSTANT HARMONIC EXCITATION

Consider a system consisting of spring, mass and damper as shown in Fig.,. Let the system is acted upon by an external periodic (*i.e.* simple harmonic) disturbing force,, $F = F_o sin\omega t$



inertia force= $m\dfrac{d^2x}{dt^2}$

Damping force= $c\dfrac{dx}{dt}$

Spring force= sx

$Impresed\ Force, F = F_o sin\omega t$

$$m\frac{d^2x}{dt^2} + [c\frac{dx}{dt} + sx - F_o sin\omega t] = 0$$

$$x_p = \frac{F_o}{\sqrt{(s - m\omega^2)^2 - (c\omega)^2}} sin(\omega t - \varphi)$$

$$x_{max} = \frac{F_o/s}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$r = \frac{\omega}{\omega_n}$$

When no damper ζ = 0

$$\chi_{max} = \frac{F_o/s}{(1 - r^2)}$$

At resonance $\omega = \omega_n$ r =1

$$\chi_{max} = \frac{F_o}{2\zeta s}$$

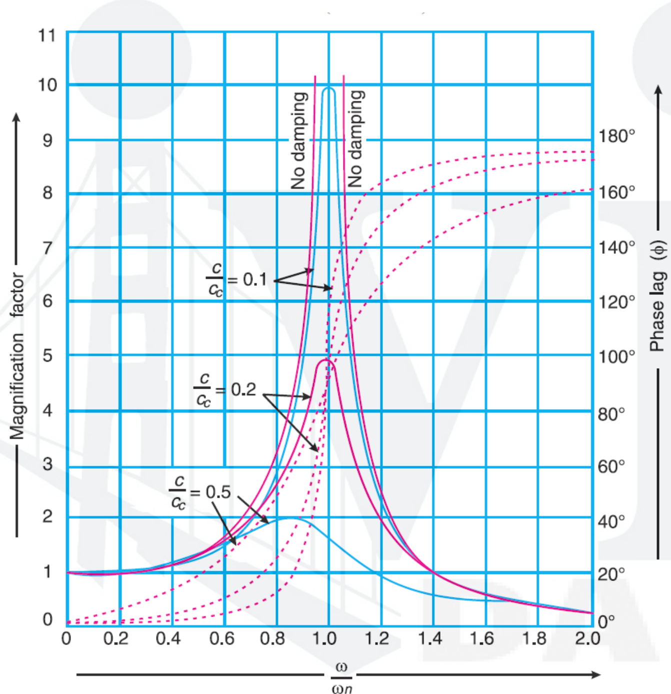
To find Phase lagϕ

It depends on the frequency of excitation and the properties of the spring-mass system.

$$\Phi = \tan^{-1}(\frac{2\zeta r}{1 - r^2})$$

Magnification Factor or Dynamic Magnifier

It is the ratio of ***maximum displacement of the forced vibration (xmax) to the deflectiondue to the static force F(xo).*** We have proved in the previous article that the maximum displacementor the amplitude of forced vibration,



$$x_{max} = \frac{F_o/S}{\sqrt{(1-r^2)^2 - (2\zeta r)^2}} = \frac{x_o}{\sqrt{(1-r^2)^2 - (2\zeta r)^2}}$$

$$M.F = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{(1-r^2) - (2\zeta r)^2}}$$

When no damper ζ = 0

$$M.F = \frac{x_{max}}{x_o} = \frac{1}{1-r^2}$$

At resonance $\omega = \omega_n$ r =1

$$M.F = \frac{1}{2\zeta}$$

A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine has a mass of 20 kg and move through a vertical stroke of 150 mm with simple harmonic motion. A dashpot is provided whose damping resistance is directly proportional to the velocity and amounts to 1.5 kN per metre per second. Considering that the steady state of vibration is reached ; determine : 1. the amplitude of forced vibrations, when the driving shaft of the engine rotates at 480 r.p.m., and 2. the speed of the driving shaft at which resonance will occur.

Given. m = 300 kg; δ = 2 mm = 2 × 10–3 m ; m_R= 20 kg ; l = 150 mm = 0.15 m ;

c = 1.5 kN/m/s = 1500 N/m/s ; N = 480 r.p.m. or ω = 2π×480 / 60 = 50.3 rad/s

Solution

1. Amplitude of the forced vibrations

Stiffness of the frame S = m.g / δ = 300 × 9.81/2 × 10–3 = 1.47 × 10^6 N/m

The length of stroke (l) = 150 mm = 0.15 m,

Radius of crank r = l / 2 = 0.15 / 2 = 0.075 m

F = $m_R.\omega^2$ .r = 20 $(50.3)^2$ 0.075 = 3795 N

$$x_{max} = \frac{F}{\sqrt{c^2\omega^2 - (s - m\omega^2)^2}} = \frac{F_O/S}{\sqrt{(2\zeta r)^2 - (1 - r^2)^2}}$$

$$x_{max} = \frac{3795}{\sqrt{1500^2 50.3^2 - (1.47 \times 10^6 - 300 \times 50.3^2)^2}} = 5.27 \times 10^{-3} m$$

$$x_{max} = 5.27\, mm$$

2. Speed at resonance occurs ($\omega = \omega_n$)

$$\omega = \omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{1.47 \times 10^6}{300}} = 70\, rad/s$$

$$N = \frac{\omega \times 60}{2\pi} = \frac{70 \times 60}{2\pi} = 668.4\, rpm$$

FORCING CAUSED BY UNBALANCE

$$x_{max} = \frac{m_u\omega^2 e}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

$$\frac{x_{max}}{\frac{m_u e}{m}} = \frac{r^2}{\sqrt{(1 - r^2)^2 - (2\zeta r)^2}}$$

A single cylinder vertical petrol engine of total mass of 200kg is mounted upon a steel chasis frame. The vertical static deflection of the frame is 2.4mm due to the weight of the engine. The mass of the reciprocating parts is 18kg and the stroke of the piston is 160mm with S.H.M. If dashpot of damping coefficient of 1N/mm/s is used to dampen the vibrations, calculate at steady state (i) the amplitude of forced vibrations at 500 rpm engine speed, and (ii) the speed of the driving shaft at which resonance will occur.

Given m = 200 kg; δ = 2.4 mm = 0.0024 m ; m_u= 9 kg ; L = 160 mm = 0.16 m ;

c = 1 kN/m/s = 1000 N/m/s ; N = 500 r.p.m. or ω = 2π×500 / 60 = 52.36 rad/s

Solution

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.0024}} = 63.93\ rad/s$$

$$r = \frac{\omega}{\omega_n} = \frac{52.36}{63.93} = 0.819$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{1000}{2 \times 200 \times 63.93} = 0.0391$$

$$e = \frac{Stroke}{2} = \frac{0.160}{2} = 0.08\ m$$

(i) The amplitude of forced vibrations

$$\frac{x_{max}}{\frac{m_u e}{m}} = \frac{r^2}{\sqrt{(1 - r^2)^2 - (2\zeta r)^2}}$$

$$\frac{x_{max}}{\frac{9 \times 0.08}{200}} = \frac{0.819^2}{\sqrt{(1 - 0.819^2)^2 - (2 \times 0.039 \times 0.819)^2}}$$

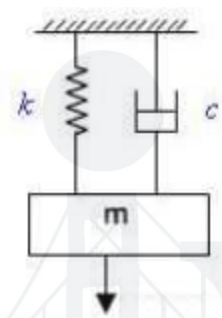
$$x_{max} = 7.2 \times 10^{-3}\ m = 7.2\ mm$$

(ii) the speed of the driving shaft at which resonance will occur ($\omega = \omega_n$)

$$\omega = \omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.0024}} = 63.93\ rad/s$$

$$N = \frac{\omega \times 60}{2\pi} = \frac{63.93 \times 60}{2\pi} = 610.5\ rpm$$

FORCED VIBRATIONS DUE TO EXCITATION OF THE SUPPORT



$$\frac{x_{max}}{Y} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 - (2\zeta r)^2}}$$

The support of a spring mass system is vibrating with an amplitude of 6mm and a frequency of 20 Hz. If the mass is 1.1 kg and the spring has a stiffness of 2000N/m, determine the amplitude of vibration of the mass. What amplitude will result if a damping factor of 0.25 is included in the system.

Given Y =6mm = 0.06m, f = 20 Hz, m=1.1kg, S = 2000N/m

Solution

$$\omega = 2\pi f = 2\pi \times 20 = 125.66\ rad/s$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{2000}{1.1}} = 42.64 \frac{rad}{s}$$

$$r = \frac{\omega}{\omega_n} = \frac{125.66}{42.64} = 2.947$$

$$\frac{x_{max}}{Y} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 - (2\zeta r)^2}}$$

$\zeta = 0,$

$$\frac{x_{max}}{Y} = \frac{1}{1 - r^2}$$

$$x_{max} = 7.807 \times 10^{-4}\, m = 0.781 mm$$

$\zeta = 0.25,$

$$\frac{x_{max}}{Y} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 - (2\zeta r)^2}}$$

$$x_{max} = 1.367 \times 10^{-3}\, m = 1.367 mm$$

VIBRATION ISOLATION

The process of reducing the vibrations of machines and hence reducing the transmitted force to the foundation using vibration isolating materials is called vibration isolation.

Isolating Materials

- Rubber
- Felt
- Cork
- Metallic springs

TRANSMISSIBILITY

$$\text{Force transmissibility} = \frac{\text{Force transmitted to the foundation}}{\text{Force applied on the system}}$$

$$\varepsilon = \frac{F_T}{F_O} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

The mass of an electric motor is 120 kg and it runs at 1500 r.p.m. The armature mass is 35 kg and its C.G. lies 0.5 mm from the axis of rotation. The motor is mounted on five springs of negligible damping so that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs. Determine 1. stiffness of each spring; 2. dynamic force transmitted to the base at the operating speed; and 3. natural frequency of the system.

Given m1 = 120 kg ; m2 = 35 kg; r = 0.5 mm = 5 × 10–4 m; ε = 1 / 11; N = 1500 r.p.m. or

ω = 2π × 1500 / 60 = 157.1 rad/s ;

Solution.

1. Stiffness of each spring

transmissibility ratio $\varepsilon = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$

$$\frac{1}{11} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\omega_n = 45.35\ rad/s$$

$$\omega_n = \sqrt{\frac{S}{m}}$$

$$S = 246840\ N\ m$$

Since these are five springs, therefore stiffness of each spring

$$= \frac{246840}{5} = 49368\ N\ m$$

2. Dynamic force transmitted to the base at the operating speed (i.e. 1500 r.p.m. or 157.1 rad/s)

We know that maximum unbalanced force on the motor due to armature mass,

$$F = m_2\omega^2 r = 35 \times (157.1)^2 \times 5 \times 10^{-4} = 432\ N$$

$$\varepsilon = \frac{F_T}{F}$$

$$F_T = \varepsilon F = \frac{1}{11} \times 432 = 39.27\ N$$

3. Natural frequency of the system

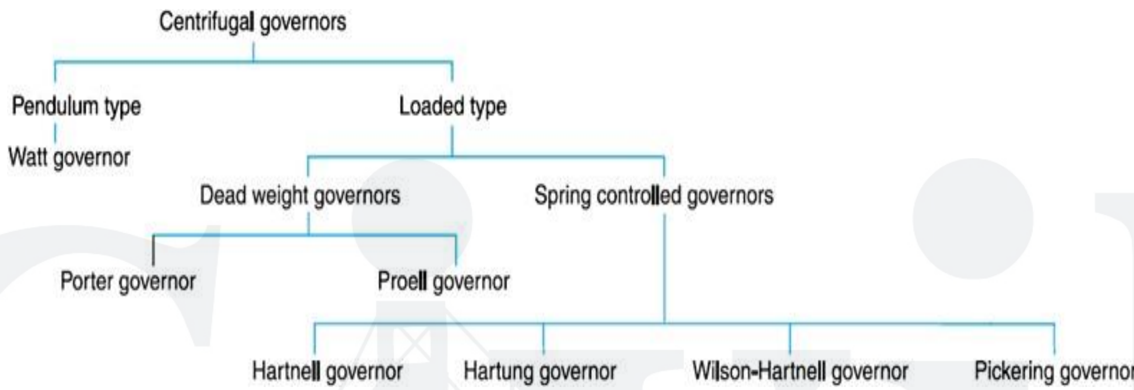
We have calculated above that the natural frequency of the system,

ω_n =45.35 rad/s

UNIT V MECHANISM FOR CONTROL

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

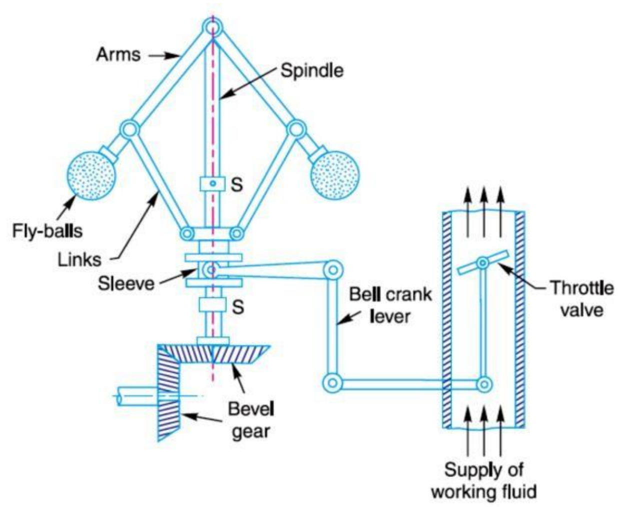
Introduction



The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

Centrifugal governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force. It consists of two balls of equal mass, which are attached to the arms as shown in Fig. These balls are known as governor balls or fly balls. The balls revolve with a spindle, which is driven by the engine through bevel gears. The



upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls.

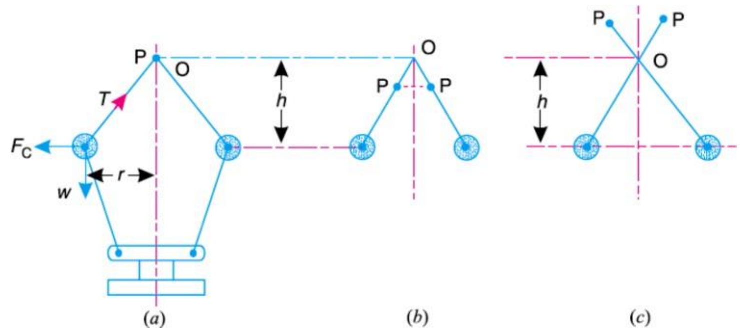
When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Terminology used in Governors

- Height of a governor
- Radius of rotation
- Equilibrium speed
- Sleeve lift
- Centrifugal force
- Controlling force
- Govornor effort
- Power of governor

Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the



governor may be connected to the spindle in the following three ways : (i) The pivot P, may be on the spindle axis as shown in Fig. (a). (ii). The pivot P, may be offset from the spindle axis and the arms when produced intersect at O, as shown in Fig. (b). (iii). The pivot P, may be offset, but the arms cross the axis at O, as shown in Fig. (c).

$$h = \frac{g}{\omega^2} = \frac{895}{N^2}$$

Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m. Also find the change in vertical height when its speed increases to 61 r.p.m.

Solution

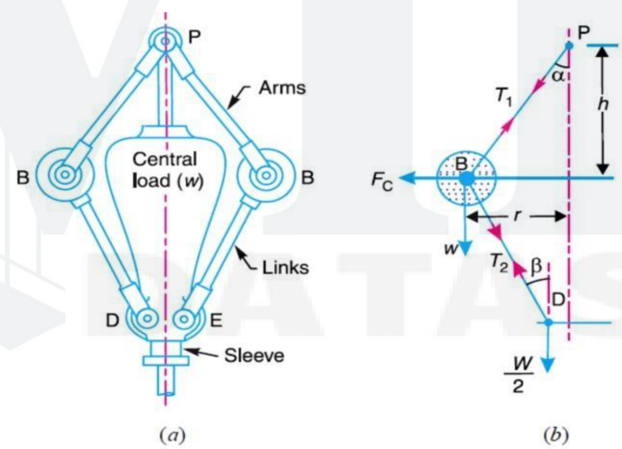
$$h_1 = \frac{895}{{N_1}^2} = \frac{895}{60^2} = 0.248\, m$$

$$h_2 = \frac{895}{{N_2}^2} = \frac{895}{61^2} = 0.24\, m$$

$$Change\ in\ vertical\ height = h_1 - h_2 = 0.248 - 0.24 = 0.008\, m = 8mm$$

Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.



$$h = \frac{m + \frac{M}{2}(1+q)}{m} \text{x} \frac{895}{N^2}$$

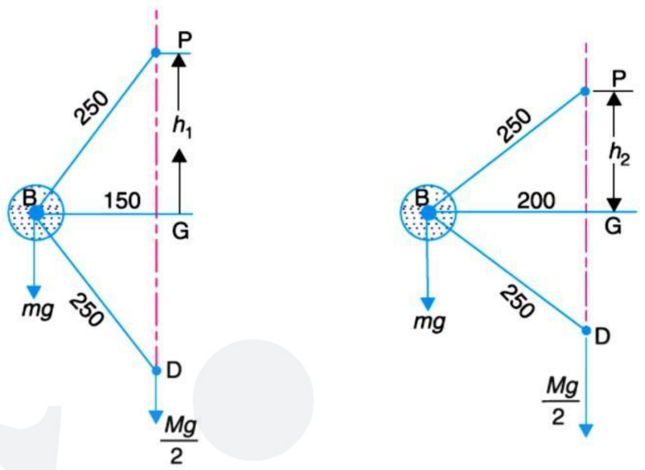
Effect of friction on Governor

$$h = \frac{mg + [\frac{Mg + F}{2}](1+q)}{mg} \text{x} \frac{895}{N^2}$$

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Given : BP = BD = 250 mm = 0.25 m ; m = 5 kg ; M = 15 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m

Solution.



(*a*) Minimum position. (*b*) Maximum position.

Minimum speed when r_1 = BG = 0.15 m

Let N_1 = Minimum speed.

height of the governor, $h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2$ m

$$h = \frac{m + \frac{M}{2}}{m} \text{x} \frac{895}{N^2}$$

$$N_1^{\ 2} = \frac{5 + \frac{15}{2}}{2} \text{x} \frac{895}{0.2} = 133.8\, rpm$$

Maximum speed when r_2 = BG = 0.2 m

Let N_2 = Maximum speed.

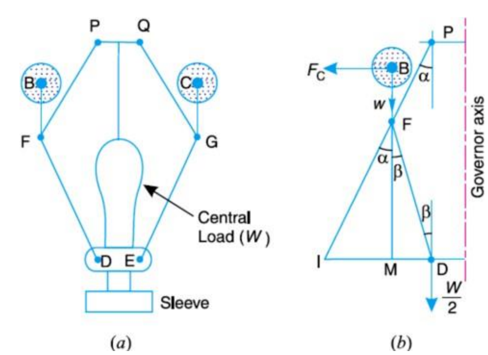
height of the governor, $h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15$ m

$$h = \frac{m + M}{m} \text{x} \frac{895}{N^2}$$

$$N_2^{\ 2} = \frac{5+15}{5} \text{x} \frac{895}{0.15} = 154.5\, rpm$$

Range of speed, = $N_2 - N_1$ = 154.4 – 133.8 = 20.7 r.p.m.

Proell Governor



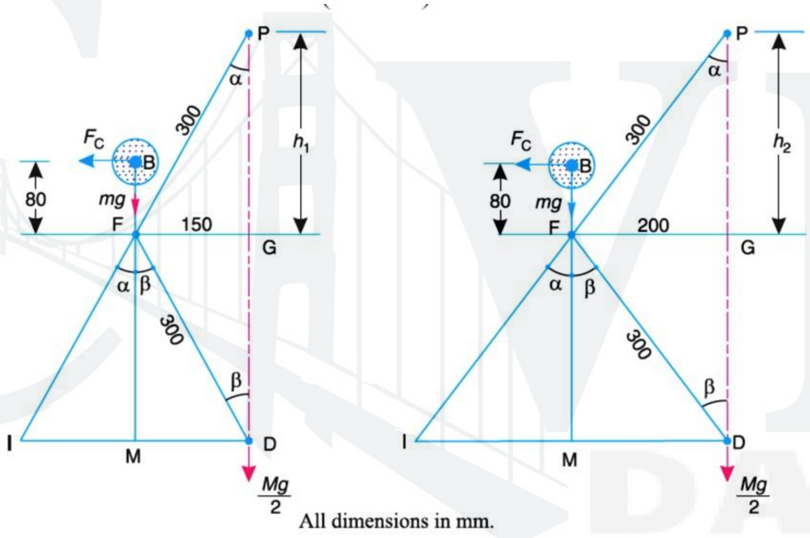
The Proell governor has the balls fixed at B and C to the extension of the links DF and EG, as shown in Fig. 18.12 (a). The arms FP and GQ are pivoted at P and Q respectively.

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Given : PF = DF = 300 mm ; BF = 80 mm ; m = 10 kg ; M = 100 kg ;r_1 = 150 mm; r_2 = 200 mm

Solution



(a) Minimum position. (a) Maximum position.

Minimum speed when r_1 = BG = 0.15 m

Let N_1 = Minimum speed.

height of the governor, h_1 = PG = $\sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.3)^2 - (0.15)^2} = 0.26$m

FM = GD = PG = 260 mm = 0.26 m

∴ BM = BF + FM = 80 + 260 = 340 mm = 0.34 m

$$h = \frac{FM}{BM}\left[\frac{m+M}{m} \times \frac{895}{N^2}\right]$$

$$N_1^{\ 2} = \frac{0.26}{0.34}\left[\frac{10+100}{10} \text{x} \frac{895}{0.26}\right] = 170\ rpm$$

Maximum speed when r_2 = BG = 0.2 m

Let N_2 = Maximum speed.

height of the governor, h_2 = PG = $\sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.3)^2 - (0.2)^2} = 0.224$ m

FM = GD = PG = 224 mm = 0.224 m

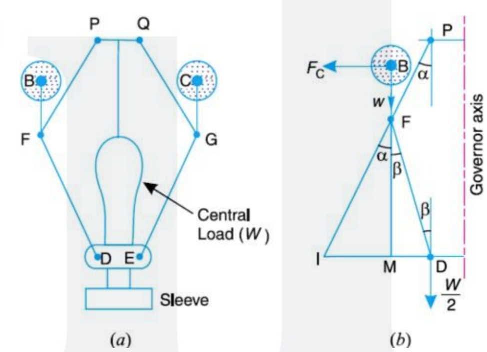
∴ BM = BF + FM = 80 + 224 = 304 mm = 0.304 m

$$h = \frac{m+M}{m} \text{x} \frac{895}{N^2}$$

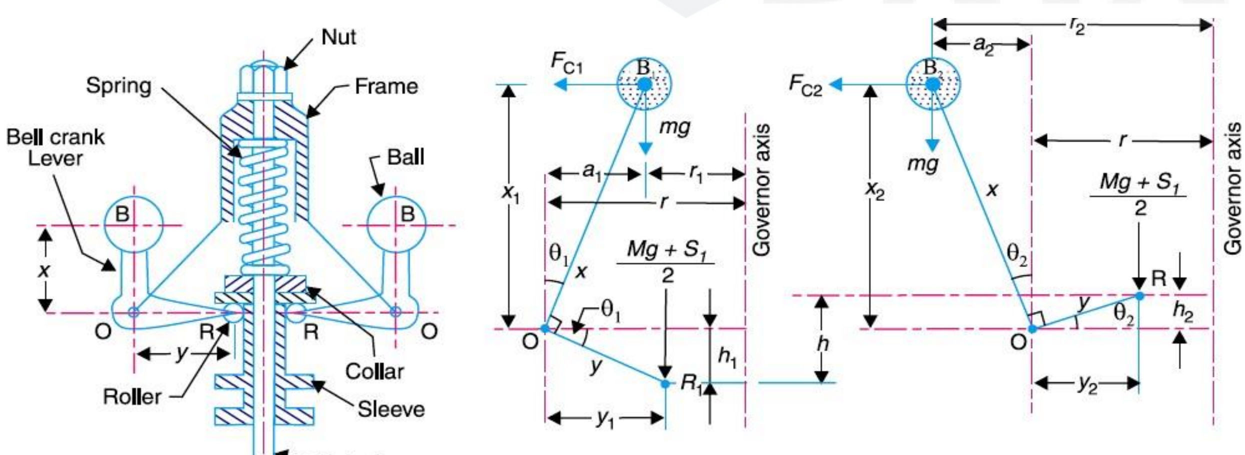
$$N_2^{\ 2} = \frac{10+100}{10} \text{x} \frac{895}{0.224} = 180\ rpm$$

Range of speed, = $N_2 - N_1$ = 180 – 170 = 10 r.p.m.

Hartnell Governor



A Hartnell governor is a spring loaded governor as shown in Fig. 18.18. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



(a) Minimum position. (b) Maximum position.

Spring Force

$$Mg + S = 2\left(\frac{a}{b}\right) F_c$$

At Minimum

$$Mg + S_1 = 2\left(\frac{a}{b}\right) F_{c1}$$

At Maximum

$$Mg + S_2 = 2\left(\frac{a}{b}\right) F_{c2}$$

$$S_1 - S_2 = 2\left(\frac{a}{b}\right)(F_{c1} - F_{c2})$$

Sleeve Lift

$$x = \left(\frac{b}{a}\right)(r_2 - r_1)$$

Spring Stiffness

$$s = \frac{S_2 - S_1}{x}$$

Intital Compression of spring (δ)

$$\delta = \frac{S_1}{s}$$

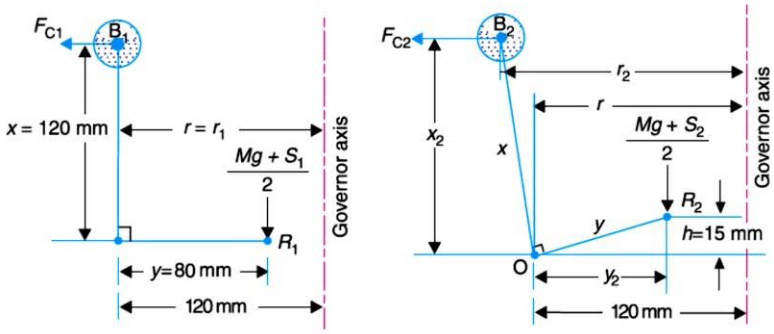
A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : 1. loads on the spring at the lowest and the highest equilibrium speeds, and 2. stiffness of the spring

Given : N1 = 290 r.p.m. or $\omega_1 = 2\pi \times 290/60 = 30.4$ rad/s ; N2 = 310 r.p.m. or $\omega_2 = 2\pi \times 310/60 = 32.5$ rad/s ; h = 15 mm = 0.015 m ; y = 80 mm = 0.08 m ; x = 120 mm = 0.12 m ; r = 120 mm = 0.12 m ; m = 2.5 kg

1. Loads on the spring at the lowest and highest equilibrium speeds

$$r = r_1 = 120 \text{ mm} = 0.12 \text{ m}$$

$$F_{C1} = (\omega_1)^2 r_1 = 2.5 \times (30.4)^2 \times 0.12 = 277\ N$$



(a) Lowest position. (b) Highest position.

r_2 = Radius of rotation at N_2 = 310 r.p.m.

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425$$

$$F_{C2} = (\omega_2)^2 r_2 = 2.5(32.5)^2 0.1425 = 376\ N$$

$$Mg + S_1 = 2F_{C1}\frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831N$$

$$S_1 = 831\ N$$

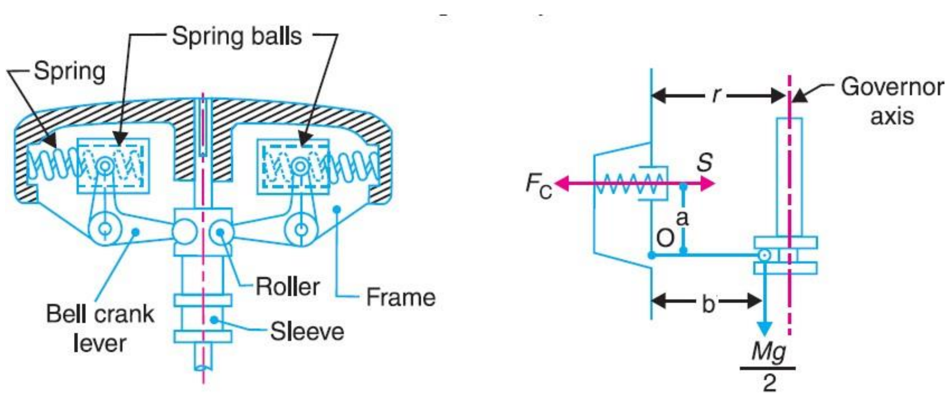
$$Mg + S_2 = 2F_{C2}\frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128N$$

$$S_2 = 1128\ N$$

Stiffness of the spring

$$S = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8\ N/mm$$

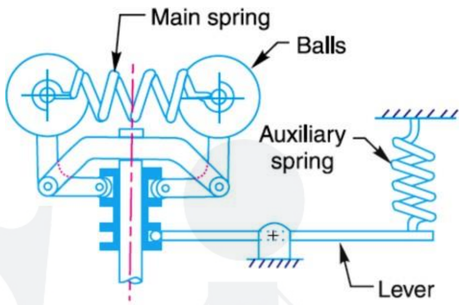
Hartung Governor



A spring controlled governor of the Hartung type is shown in Fig. In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

$$F_C \times a = Sa + \frac{Mg}{2} \times b$$

Wilson Hartnell Governor



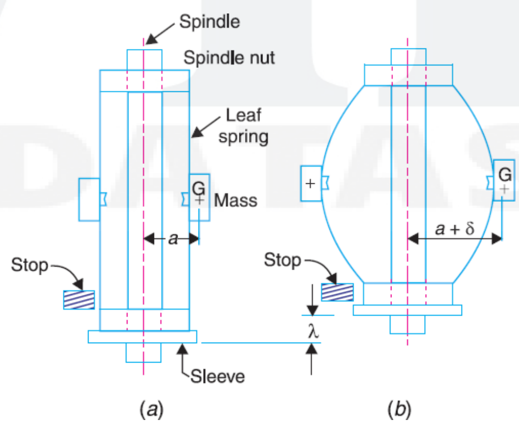
Wilson-Hartnell governor.

$$(F_C - P) \times a = \frac{Mg + s(\frac{y}{x})}{2} \times b$$

$$4\, s_b + \frac{s_a}{2} [\frac{b}{a} \times \frac{y}{x}]^2 = \frac{F_{c1} - F_{c2}}{r_2 - r_1}$$

Pickering Governor

A Pickering governor is mostly used for driving gramophone. It consists of three straight leaf springs arranged at equal angular intervals round the spindle. Each spring carries a weight at the centre. The weights move outwards and the springs bend as they rotate about the spindle axis with increasing speed.



$$W = F_C = m\omega^2(a + \delta)$$

$$\delta = \frac{m\omega^2(a + \delta)l^3}{192EI}$$

Performance Characteristics of Governors

Sensitiveness

The sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

$$\text{Sensitiveness} = \frac{\text{Mean Speed}}{\text{Range of Speed}} = \frac{N}{N_2 - N_1}$$

Stability

A governor is said to be stable when for every speed within the working range there is a definite configuration.

Isochronism

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

For isochronism, range of speed should be zero i.e. $N_2 – N_1 = 0$ or $N_2 = N_1$ therefore $h_1 = h_2$, which is impossible in case of a Porter governor. Hence a Porter governor cannot be isochronous.

For isochronism

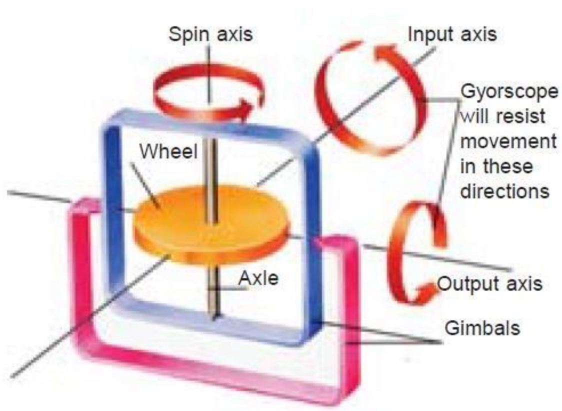
Hunting

A governor is said to be hunt if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place.

Gyroscopic Couple

When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as active force

When a body, itself, is moving with uniform linearvelocity along a circular path, it is subjected to the centrifugal force radially outwards. This centrifugal force is called reactive force.



Gyroscopic Couple

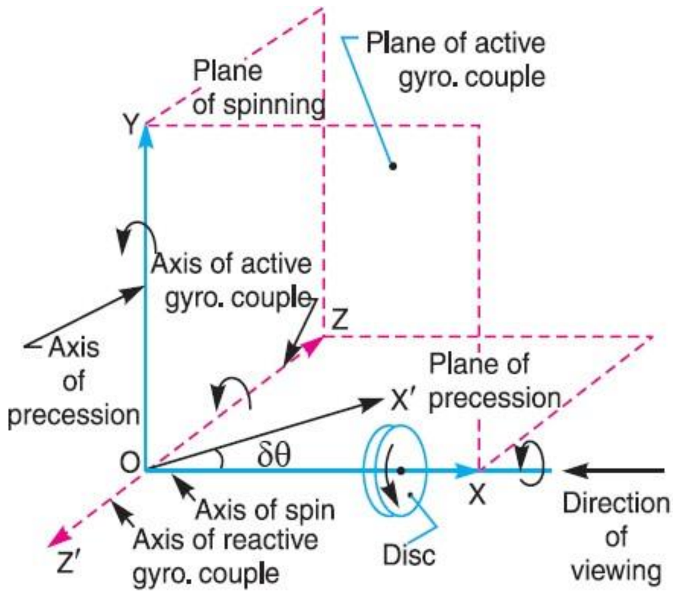
Consider a disc spinning with an angular velocity ω rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig. Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity ωP rap/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

Let , I = Mass moment of inertia of the disc about OX, and

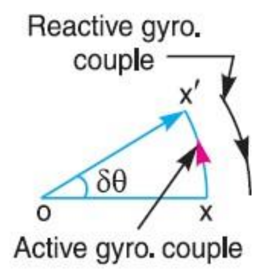
ω = Angular velocity of the disc.

∴ Angular momentum of the disc = I.ω

Since the angular momentum is a vector quantity, therefore it may be represented by the vector ox → , as shown in Fig. The axis of spin OX is also rotating anticlockwise when seen from the top about the axis OY. Let the axis OX is turned in the plane XOZ through a small angle δθ radians to the position OX ′ , in time δt seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector ox′.



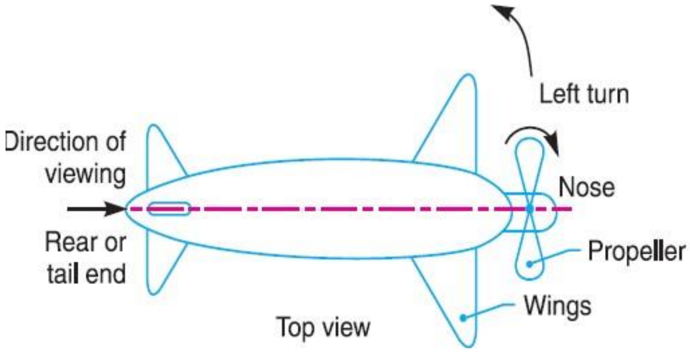
(a)

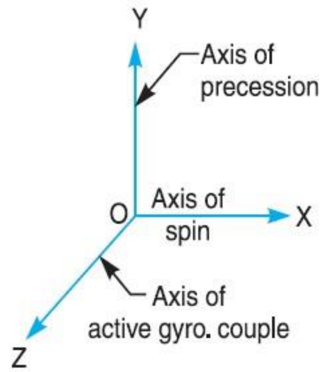


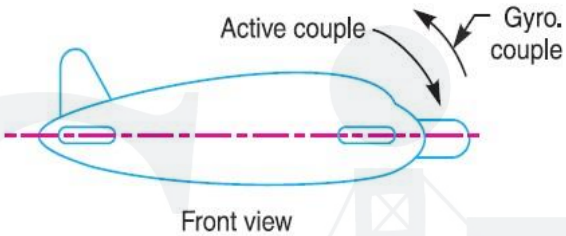
(b)

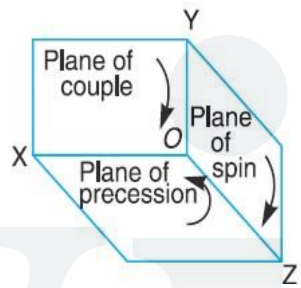
Rate of change of angular momentum . $= I\omega \frac{\delta\theta}{\delta t} = I\omega\omega_p$

Effect of the Gyroscopic Couple on an Aeroplane









Gyroscopic couple acting on the aeroplane,

$$C = I\omega\omega_p$$

When viewing from rear or tail end

Turning direction	Engine rotation direction	Gyroscopic couple direction	Effect on plane
Left turn (-ve)	Anticlockwise (-ve)	(-ve) x (-ve) = (+ve) (Clockwise)	Dip the nose & raise the tail
Right turn (+ve)	Anticlockwise (-ve)	(+ve) x (-ve) = (-ve) (Anticlockwise)	Dip the tail & raise the nose
Left turn (-ve)	Clockwise (+ve)	(-ve) x (+ve) = (-ve) (Anticlockwise)	Dip the tail & raise the nose
Right turn (+ve)	Clockwise (+ve)	(+ve) x (+ve) = (+ve) (Clockwise)	Dip the nose & raise the tail

An aero plane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ; N = 2400 r.p.m. or ω = 2π × 2400/60 = 251 rad/s

Solution

$$I = mk^2 = 400 \times (0.3)^2 = 36\ kg - m^2$$

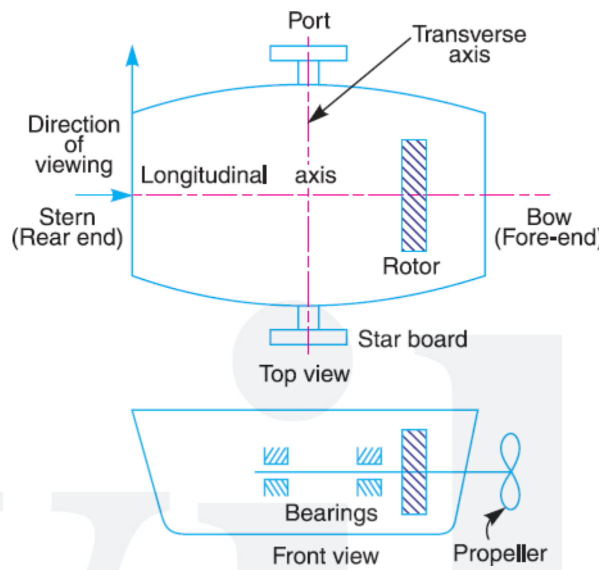
$$\omega_p = \frac{v}{R} = \frac{55.6}{50} = 1.11 rad/s$$

$$C = I\omega\omega_P = 36 \times 251 \times 1.11 = 100.46\ N - m$$

when the aero plane turns towards left, the effect of thegyroscopic couple is to lift the nose upwards and tail downward.

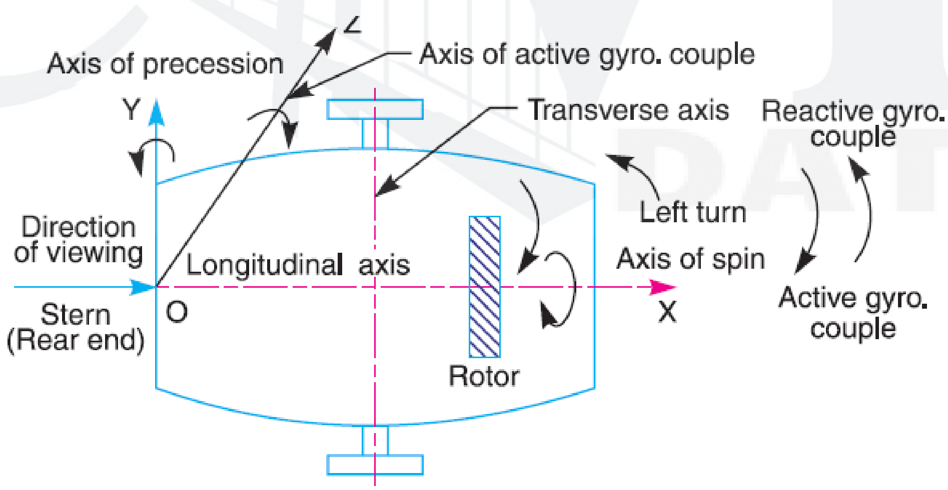
Terms Used in a Naval Ship

The top and front views of a naval ship are shown in Fig. The fore end of the ship is called bow and the rear end is known as stern or aft. The left hand and right hand sides of the ship, when viewed from the stern are called port and star-board respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:



1. Steering, 2. Pitching, and 3. Rolling.

Effect of Gyroscopic Couple on a Naval Ship during Steering



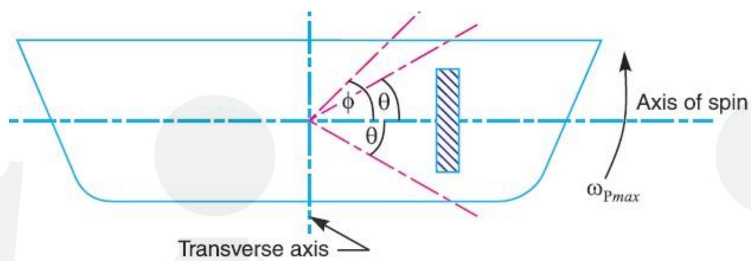
1. When the ship steers to the right under similar conditions as discussed above, the effect of the reactive gyroscopic couple, will be to raise the stern and lower the bow.

2. When the rotor rates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to lower the bow and raise the stern.

3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.

4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to raise the stern and lower the bow.
5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.
6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn is similar as discussed above.

Effect of Gyroscopic Couple on a Naval Ship during Pitching



The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Given: m = 8 t = 8000 kg ; k = 0.6 m ; N = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ;
v = 100 km/h = 27.8 m/s ; R = 75 m

Solution.

mass moment of inertia of the rotor, $I = m.k^2 = 8000\,(0.6)^2 = 2880$ kg-m^2
and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{27.8}{75} = 0.37\,rad/s$$

$$C = I\omega\omega_P = 2880 \times 188.5 \times 0.37 = 200866\,N-m = 200.866\,kN-m$$

when the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

A four-wheeled trolley car of mass 2500 kg runs on rails, which are 1.5 m apart and travels around a curve of 30 m radius at 24 km / hr. The rails are at the same level. Each wheel of the trolley is 0.75 m in diameter and each of the two axles is driven by a motor running in a direction opposite to that of the wheels at a speed of five times the speed of rotation of the wheels. The moment of inertia of each axle with gear and wheels is 18 kg-m2. Each motor with shaft and gear pinion has a moment of inertia of 12 kg-m2. The centre of gravity of the car is 0.9 m above the rail level. Determine the vertical force exerted by each wheel on

the rails taking into consideration the centrifugal and gyroscopic effects. State the centrifugal and gyroscopic effects on the trolley.

Given : m = 2500 kg ; x = 1.5 m ; R = 30 m ; v = 24 km/h = 6.67 m/s ; d_W = 0.75 m or r_W = 0.375 m ; G = ω_E/ω_W = 5 ; I_W = 18 kg-m2 ; I_E = 12 kg-m2 ; h = 0.9 m

Solution

Road reaction over each wheel = W/4 = m.g/4 = 2500 × 9.81/4 = 6131.25 N

$$\omega_W = \frac{v}{r_w} = \frac{6.67}{0.375} = 17.8\ rad/s$$

$$\omega_P = \frac{v}{R} = \frac{6.67}{30} = 0.22\ rad/s$$

Gyroscopic couple due to one pair of wheels and axle,

$$C_W = 2I_W\omega_W\omega_P = 2 \times 18 \times 17.8 \times 0.22 = 141\ N-m$$

Gyroscopic couple due to one pair of wheels and axle,

$$C_E = 2I_E\omega_E\omega_P = 2I_EG\omega_W\omega_P = 2 \times 12 \times 5 \times 17.8 \times 0.22 = 470\ N-m$$

Gyroscopic couple, C = $C_W - C_E$ = 141 – 470 = – 329 N-m

$$\frac{P}{2} = \frac{C}{2x} = \frac{329}{2 \times 1.5} = 109.7\ N$$

$$F_C = \frac{mv^2}{R} = \frac{2500 \times (6.67)^2}{30} = 3707\ N$$

Overturning couple,

$$C_o = F_C h = 3707 \times 0.9 = 3336.3\ N-m$$

$$\frac{Q}{2} = \frac{C_O}{2x} = \frac{3336.3}{2 \times 1.5} = 1112.1\ N$$

vertical force exerted on each outer wheel,

$$P_O = \frac{W}{4} - \frac{P}{2} + \frac{Q}{2} = 6131.25 - 109.7 + 1112.1 = 7142.6\ N$$

vertical force exerted on each inner wheel,

$$P_I = \frac{W}{4} + \frac{P}{2} - \frac{Q}{2} = 6131.25 + 109.7 - 1112.1 = 5128.8\ N$$